Agent-Based Modeling in Economics and Finance: Past, Present, and Future[∇]

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Abstract

Agent-based modeling is a novel computational methodology for representing the behavior of individuals in order to study social phenomena. The approach is now exponentially growing in many fields. The history of its use in economics and finance is reviewed and disparate motivations for employing it are described. We highlight the ways agent modeling may be used to relax conventional assumptions in economic models. Typical approaches to calibrating and estimating agent models are surveyed. Software tools for agent modeling are assessed. Special problems associated with large-scale and data-intensive agent models are discussed. Our conclusions emphasize the strengths of this new approach while highlighting several challenges and important open problems, both conceptual and technological, which may usefully serve as milestones for future progress in agent computing within economics and finance, specifically, and the social sciences more broadly.

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I Introduction: What is Agent-Based Modeling?

At the dawn of the digital computation era, economists, operations researchers, game theorists, and other social scientists were among the early adopters of the new technology (e.g., Mirowski, 2001). Early uses included the manipulation large matrices in input-output models (Leontief, 1951), statistical analyses of economic data (Brown, Houthakker and Prais, 1953), estimation of macro models from aggregate data (Klein and Goldberger, 1955), linear programming (Dorfman, 1951, Dorfman, Samuelson and Solow, 1958) and other mathematical programming activities (Kemeny, Morgenstern and Thompson, 1956), numerical solution of otherwise intractable microeconomic models (Cohen and Cyert, 1961), and even theorem proving (Newell and Simon, 1956). It is also the case that *novel* uses of digital computing—beyond numerical analysis, statistical methods, and mathematical logic—soon appeared. Specifically, Orcutt and co-workers (1961) pioneered *microsimulation* models, a type of stochastic simulation that remains in use today, primarily for policy purposes (e.g., Bergmann, Eliasson and Orcutt (1980), Urban-Brookings tax model). At about this same time the so-called *Carnegie School* invigorated the theory of the firm via computational models of intra-firm (organizational) behavior (Cyert and March, 1963). *System dynamics* (*SD)* tools were developed at MIT in the 1950s and '60s (Forrester, 1958), primarily as a way of modeling aggregate stocks and flows of various socially-relevant variables, often from a policy perspective.[1] Some of these new uses of digital computation fell under the broad category of *simulation*, and within operations research (*OR*) a methodology for *discrete event simulation* (e.g., Conway, Johnson and Maxwell, 1959) arose for analyzing and designing business processes. The post World War II era also gave birth to *artificial intelligence* (*AI*), a major thematic area of computer science, with strong cross-over into psychology and the behavioral sciences via the birth of cognitive science (Newell and Simon, 1972).

Today there is a computational technology diffusing through economics and finance, specifically, and the social sciences more broadly, that did not exist until more recently. Agent-based computing goes by many names and acronyms depending on the field in which it is employed—*agent-based modeling* (*ABM*) in most of the social sciences, *multi-agent systems* (*MAS*) in computer science, *individual-based modeling* (*IBM*) in ecology—and refers to a class of computational techniques that have proven useful as a way to represent individual behavior for purposes of studying social phenomena. Models of this type feature a population of software objects called *agents*, which are typically heterogeneous and situated in an economic or social environment. The individual agents are given explicit rules of behavior, which can be quite general—as in 'seek greater utility'—or very specific (e.g., 'lower prices by *5%* if inventory exceeds target levels'). The agents interact directly with one another through social, spatial, or physical networks that are either exogenously specified or endogenously generated. Such models may produce more or less conventional agent-level equilibria (e.g., Nash equilibria), or can yield perpetual dynamics at the micro-level as agents constantly adjust their behaviors. Importantly, the aggregate level is not explicitly pre-specified. Rather it *emerges* from the myriad interactions of the agents. Economists who use these methods often call what they

[1] Certain *SD* models—e.g., the *Limits to Growth* study of the Club of Rome (Meadows, Meadows, Randers and Behrens, 1972)—proved controversial by forecasting the rapid depletion of Earth's resources unless growth was severely attenuated, a conclusion widely dismissed for many reasons (Beckerman, 1972), including because there were no prices in the model (Ridker, 1973)! Since then *SD* models have usually had narrower focus (e.g., Sterman, 2000).

do *agent-based computational economics* (*ACE*, e.g., see the *Handbooks of Economics* edited by Tesfatsion and Judd (2006) and Hommes and LeBaron (2018)), and we shall use this phrase and acronym interchangeably with *ABM*.[2] This survey covers the use of computational agents in economics and finance over the past 25 years, reviewing what has been accomplished, describing the current state-of-the-art, and looking to future prospects, while assessing potential bottlenecks to further progress.[3]

A *ABM* as Computationally-Enabled Economics, from the Bottom Up

Essentially all of the sciences are today being revolutionized by digital computation, whether by generating numerical solutions to analytically intractable mathematical specifications, by enriching those specifications through relaxation of the most unrealistic aspects of them, or by systematically integrating newly available data into models. In biology, whole cells are being modeled (Karr et al., 2012), consisting of hundreds of thousands of chemical reactions. The role of all genes in large-scale regulatory (Zhu et al., 2008) and signaling (Pawson, 1995) networks are routinely analyzed. Brain models consisting of billions of digital neurons are now possible (Markram, 2006, 2012), with the hope of representing cognition in biological terms. In chemistry, medically-significant compounds can be modeled computationally (Lewars, 2011), in advance of being synthesized in the lab, and their specificity and ultimate efficacy estimated. In fluid mechanics, analytically intractable turbulent flows can now be rendered computationally with great verisimilitude, with eddies and vortices 'emerging' from the local conditions in the flow fields (Hoffman and Johnson, 2007). In astrophysics and planetary science large-scale computational models play an essential role in explaining everything from the dynamics of galaxies (Norman et al., 1996, Binney and Tremaine, 2008) to how Earth's moon came to be the way it is (Canup and Asphaug, 2001, Canup, 2012). In atmospheric physics and oceanography, in the guise of weather and climate science, supercomputing is an essential tool (Gneitling and Raftery, 2005, Edwards, 2010). In all of these areas the basic processes that operate at the microscopic level are reasonably well-understood, but the emergence of novel structures at higher levels of organization—turbulence, storms, consciousness—that are hard to understand conceptually, or seemingly impossible to study analytically, are yielding to computation, one of the few methodologies—often the only methodology—capable of producing scientific progress.

This is increasingly true in economics. While the mathematical content of modern economics has grown over time—for half a century the number of theorems, corollaries, lemmas, formal propositions, and so on appearing in economic journals has increased monotonically[4]—it is frequently either difficult or impossible to arrive at closed-form, analytical solutions, so resort is made to numerical methods (Varian, 1992, Amman,

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[2] Arguably, the modifier 'computational' is redundant in *ACE* if one understands 'agent' in *ABM* as referring to software agents. In this case the appropriate moniker would be *agent-based economics* (*ABE*) and when this was proposed 25 years ago it was claimed to be confusing since, ostensibly, all microeconomics is 'agent-based.' However, outside of principal-agent problems the term is used irregularly in microeconomics and game theory textbooks. For more on the 'agent' in economics see Foley (2002).

[3] This article is *not* a tutorial on how to create an agent-based model in software. A good qualitative overview of the modeling process is given by Gilbert (2008). For the details of programming agent models good textbooks now exist (Railsback and Grimm, 2011, Wilensky and Rand, 2015).

[4] As a crude measure of this we count 45 such statements in the Jan. 2016 issue of the *American Economic Review*, 38 in the Jan. 2006 issue, 20 in the Jan. 1996 number, 18 in Jan. 1986, 6 in Jan. 1976, 4 in Jan. 1966, 2 in Jan. 1956, and none in previous Jan. issues of 1946, 1936, 1926 and 1916.

Kendrick and Rust, 1996, Judd, 1998). *Numerical economics* is thus seen as a *complement* to extant economic theory (Judd, 1997). While this style of computational economics provides explicit solutions to particular problems, its ultimate utility depends on the veracity of the underlying equations being solved. Such equations are typically highly idealized, assuming continuity, smoothness, perfect arbitrage, rationality, and so on, meaning the relevance of the computed solutions to real economies is unclear.[5]

ABM is a different kind of computationally-enabled economics. Instead of starting with equations governing an economic process, derived from assumptions possibly made for analytical tractability, one normally begins building an *ABM* with a population of agents involved in the process, each being given behavioral specifications and initial states. Instead of solving equations for equilibrium one simply lets the agents interact directly with one another, the behavioral rules producing specific agent behaviors and new agent states. Successful models produce states that are relevant to the economic process being modeled. Unsuccessful models do not. Each realization of an *ABM* is a *theorem*: if agents start with certain initial states and engage in specific behaviors then after some number of interactions they will have definite new states. While *ABMs* often have pseudo-stochastic elements, each model 'run' is typically deterministic. Distributional properties of agent states can by characterized by making many runs of a specific model and allowing random seeds or other stochastic elements to vary.

As an example, consider the most basic model in all of microeconomics, the supply and demand for a single homogenous good is one market. Typically, the behavior of people in such a market is represented by a demand curve for buyers, downward sloping with quantities and prices on the horizontal and vertical axes, respectively, and a supply curve for sellers, usually upward sloping. This is depicted graphically in all microeconomics texts and studied algebraically by the time students reach intermediate micro. The figure—normally drawn as two straight lines—is often accompanied by the assertion that markets operate at the intersection point, while solving for the price-quantity pair is an exercise in high school algebra when the functions are given.

If you have taught this material perhaps you have had the experience of the occasional sharp student asking how we *know* that markets operate at the intersection point of supply-and-demand, or an engineering student asking what happens if supplies or demands are changing, or a behavioral science student wondering if there are empirical tests of the theory in laboratory settings. Happily, some of these questions have satisfactory answers—a good answer to the latter is that this basic set-up is very close to that used in early experiments (Smith, 1962) and is today a standard experimental protocol (Bergstrom and Miller, 1997). But it is also true that at least one such elementary question does *not* seem to have an elementary answer. For the existence of a supply-demand equilibrium in general is tied up with assumptions about continuity and the existence of Brouwer fixed points (Arrow and Debreu, 1954), which is a topic in real analysis and famously non-constructive. In any case, this is not easily communicated to undergraduates taking intermediate micro without some hand-waving, or invoking the intermediate value theorem for the benefit of those students who have had calculus.

A different way to respond to such queries, and a more direct assessment of how markets work, involves telling students that one of the powerful features of markets is

[5] For example, in Judd's (1998) textbook 'heterogeneous agents' are only encountered in the last chapter, and only for $N = 2$ such agents!

that no individual needs to know very much about how a market functions—participants do not have to be economists!—to get reasonable performance (Smith, 2008). That is,

1. buyers who have heterogeneous internal valuations will together create a downward sloping demand curve (Becker, 1962) , while
2. sellers with heterogeneous costs will produce an upward-sloping supply curve,
3. so if buyers pay less than their internal values and sellers try to cover costs,
4. there can result relatively high market efficiency (Gode and Sunder, 1993).

Indeed, a reasonable answer to the inquisitive student is that a different way to conceive of a market is as people (agents) engaged in local exchanges, and can be modeled exactly this way with interacting software agents (cf. Palmer et al., 1994, Epstein and Axtell, 1996, Cliff and Bruten, 1997b). One can see this by pointing a browser at Mark McBride's website here (http://www.memcbride.net/models/2014/7/11/zi-trading) and running an *ABM* of just such a market. It features a user definable number of buyers and sellers whose internal valuations are specified and from which supply and demand curves can be plotted. One immediate pedagogical benefit of giving agents heterogeneous valuations and costs is that the supply and demand curves are much less regular than what one finds in microeconomics textbooks! Hitting 'Go' one sees that trades take place and the quantity exchanged is comparable to the point estimate of the supply and demand curves, although rarely exactly the same, as shown in figure 1.

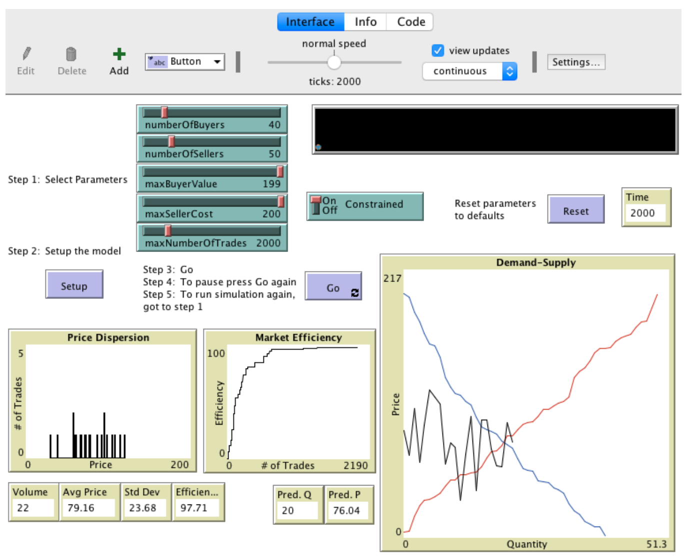

Figure 1*: Screen capture from the zero intelligence trader model of M. McBride, in NetLogo software; the red line is the supply curve, the blue line demand, and the black line represents prices paid; other figures show overall market efficiency and price dispersion; each realization of the model yields a different result*

This leads most students to the inference that the point estimate of the textbook story is just one thing that can happen, and there are other things worth exploring. By rerunning the model multiple times students see that (1) the same price and quantity never repeat, and (2) there is sufficient variability from run to run as to give the point prediction more the character of a central tendency, a kind of idealized outcome but not the only thing that can happen. Students can then be led to questions concerning what happens to the variability as the size of the market increases, or as elasticities change. Or as one transitions from local barter to auctions to posted prices (e.g., from farmers' markets to Treasury bill auctions to retail stores) is there more to markets than the simple world of two intersecting straight lines? *ABMs* of specific markets, and agent computing in economics generally, has the ability to enrich student understanding of what we are tying to teach them by treating economic phenomena as emerging from the bottom up, as a direct result of the actions and interactions of purposeful individuals. The *ABM* approach to economics turns student attention away from mathematical difficulties that are of little relevance to real markets (e.g., fixed point theorems) and towards how people behave in various market contexts. By utilizing the computer in ways that are richer than merely solving equations—by displaying model output visually, computing statistics dynamically, and permitting students to modify how models work—a whole new way of teaching and learning economics is opened up by *ABMs*.

Looking broadly at the many ways *ABMs* have grown up and evolved in economics, finance, and the wider social sciences, there is a relatively small set of features characteristic of such models. Specifically, *ABMs* always possess:

- One or more populations of agent software objects, each agent representing an individual person or group (e.g., a business firm) and having local state information (e.g., preferences, income, job, gender);
- Agent behavioral specifications that typically are conditional on the state of the agent and therefore behavior is heterogeneous across the population;
- An environment of some kind, possibly including aggregate economic variables (e.g., prices, interest rates), interaction networks, space, and so on;
- A scheme for agent updating, whether a pre-computed 'schedule,' a stochastic process that selects which agents are active at any instant, or some other mechanism providing a certain degree of agent autonomy;
- A visualization engine of some kind, to depict the activity of the agents in a way that is meaningful for assessing that the model is performing as desired;
- Data gathering and statistical analysis facilities, typically in real-time with model execution, for determining the global state of the model and the behaviors of certain sub-populations;

Additionally, most *ABMs* are run in an integrated develop environment of some kind that permits the user to analyze and debug his or her model while providing facilities for making large numbers of runs by varying parameters, archiving agent and aggregate state information, and serving as a repository for code.

It is also interesting to note what is normally *not* present in *ABMs*. Typically there are no equations that explicitly relate aggregate states to agent-level states, although certainly these exist implicitly via aggregation and accounting relations. The main point is that equations are not being 'solved' inter-agent, although it is common for individual agents to have mathematical representations of their environment as they try to decide how to behave. There are also no overt 'solution methodologies' for computing things like Nash equilibria, market-clearing prices, efficient networks, or optimal policies. (There may exist regulator agents in some models but they are typically not afforded omniscience or the ability to overtly control other agents.) An *ABM* may evolve, over time, into a configuration that resembles Walrasian equilibria, or is demonstrably a Nash equilibrium, say, but in so doing it would be a consequence of the behavioral rules the agents are following and not because it has been imposed by fiat or stipulated by the model builder simply because it is a well-understood solution concept.

The existence of this new approach to economic modeling has come at a time when results from behavioral and experimental economics offer novel ways for representing what humans actually do (Gigerenzer and Selten, 2001). Using the rich expressiveness of computer code we can create agents with just such behaviors. It is also the case that exciting developments in the network sciences (Vega-Redondo, 2007, Jackson, 2008) provide a new lens to look at decentralized interactions at the same time computational tools for network analysis and visualization have become available. Furthermore, increasing amounts of micro-data (e.g., Mian, Rao and Sufi, 2013) give us the ability to see the full range of heterogeneity present in populations and to represent this in models (Guvenen, 2011). These new capabilities may help economists accelerate research with new kinds of behavioral specifications and other abstractions in order to realize new types of models that some have called for (e.g., Kristol and Bell (1981), Kirman (1989, 2010), Hahn (1992), Stiglitz (2009), Colander et al. (2009) and Trichet (2010)).

B Potential Usefulness of *ABM* for Research in Economics and Finance

Models are composed of idealizations. In economics and finance, idealizations take the form of things like rational or representative agents, global price vectors, and Nash equilibrium. Ideal types are *not* exact statements about how the world works but rather approximations that are easy to imagine or write down or analyze. They facilitate modeling. An abstraction's value may derive from the closeness with which it approximates reality. Alternatively it may be such a gross approximation as to be empirically false but valuable nonetheless because it permits a model that is otherwise intractable to be solved. Each of the modeling abstractions we teach economics graduate students has a mixture of these features: it approximates reality to some degree while facilitating analysis. The status of these abstractions as imperfect but necessary for progress is well understood, for much of what passes for research in economics amounts, in some way or another, to relaxation of one or more of them, usually by replacing a gross idealization with something having higher fidelity. Indeed, the research literature in economics is largely composed of work that replaces one or two of the standard assumptions, generating conclusions that encompass but are different from those produced by the usual specifications. Table 1 below lists important features of economic models in the first column and standard neoclassical abstractions in the second, roughly in line with what is taught (e.g., Mas-Collel, Whinston and Green, 1995). The third column gives some of the ways the standard conceptions have been relaxed or generalized in the economics literature. We term these 'complexity economics'. While any particular entry in that column may have its own moniker—e.g., 'behavioral economics' for the fourth row—the column *as a whole* ranges over topics that are foundational to the study of *complex adaptive systems* (CAS); see Anderson, Arrow and Pines (1988), Mirowski (1996), Arthur, Durlauf and Lane (1997), Blume and Durlauf (2005), Kirman (2011).[6] There is a close relationship between *ABM* and *CAS* just as there

[6] For succinct introductions to *CAS* see Holland (1998, 2012, 2014) or Bak (1996); for a social science point-of-view there are Miller (2015) and Miller and Page (2007); Boccara (2010) is a more mathematical treatment with a natural science focus; for a computer science perspective consult Mitchell (2009) while Downey (2012) provides working *ABM* code; Krugman's (1996) take is very readable, albeit dated; Durlauf (2012) expresses skepticism of the relevance of *CAS* to economics despite earlier views to the contrary; others are optimistic (cf. Colander (2000), Potts (2000), Berry, Kiel and Elliott (2002), Kirman (2004), Rosser (2004), Axtell (2007), Farmer and Foley (2009), Farmer and Geanakoplos (2009), Rosser (2010), Holt, Rosser and Colander (2011), Gallegati and Kirman (2012), Colander and

is between nonlinear dynamics and complexity. In mathematical chaos a relatively low dimensional (usually) dynamical system has dynamics that are bound to an attractor that is neither a fixed-point equilibrium nor a limit cycle. These dynamics are complex despite the relative simplicity of the equations. With agents we have essentially the inverse situation, a very high dimensional dynamical system with perpetual adaptation and change at the micro (agent) level, but at the aggregate or macro level there emerge gross (low dimensional) patterns and regularities.

Model feature	Neoclassical conception	'Complexity economics' conception
Number of agents	representative (1, 2, infinite)	many (preferably full-scale)
Diversity of agents	homogeneous, a few 'types'	heterogeneous, idiosyncratic agents
Agent goals, objectives	static, scalar-valued utility	evolving, other-regarding
Agent behavior	rational, maximizing, brittle	purposive, adaptive, biased, myopic
Learning	individual, fictitious play	empirically-grounded, social
Information	centralized, possibly uncertain	distributed, tacit, costly to acquire
Beliefs	coordinated for free	uncoordinated, costly to coordinate
Interaction topology	equal probability, well-mixed	social networks, fixed and changing
Markets	auctioneer, global price vector	decentralized, local prices
Firms and institutions	unitary actors, production fcns	multi-agent groups and organizations
Selection	single level	multi-level, group selection
Governance	benevolent planner, median voter	self-governance, incentive problems
Temporal structure	static, impulse tests, 1-shot	dynamic, full transient paths
Source of dynamism	exogenous, outside economy	endogenous to the economy
Properties of dynamics	smooth, differentiable	irregular, volatile, heavy-tailed
Character of dynamics	Markovian, path is forgotten	path-dependent, history matters
Solution concepts	equilibrium at agent level	macro steady-state (stationarity)
Multi-level character	neglected, dual fallacies	intrinsic, macro-level emerges
Methodology	deductive, mathematical	abductive, computational
Ontology	representative agent, *max U*	ecology of interacting agents
Policy stance	designed from the top down	evolved from the bottom up

***Table 1**: Contrast between standard economic abstractions and more realistic ones*

What keeps economists from moving from the middle column to the right are a variety of conceptual, mathematical and econometric difficulties that make richer models intractable in one way or another.[7] Agent computing offers a way to explore the right column. For each row there exist *ABMs* relevant to the right column. *ABMs* are a new tool in the economist's toolbox that can be used to move from the center toward the right column. Unfettered by the mathematical strictures that constrain the kinds of economic models that can be written down and solved analytically, *ABMs* serve as a potent new methodology for accelerating progress in economics.

ABM facilitates movement towards the right column by combining the *expressiveness* of computer code with a computational methodology that does not explicitly pre-specify aggregate outcomes. Agent computing permits one to write economic specifications that have high fidelity with the real-world, whether behavioral, institutional (e.g., constitutional rules), or administrative/legal (e.g., regulatory rules). For example,

Kupers (2014), Arthur (2015), Haldane and Turrell (2018)). Earlier versions of this table appear in Axtell et al. (2016), Axtell (2017). For a similar but more evolutionary perspective see Bowles (2004: 479).

[7] In his masterful *Engine Not a Camera* Donald Mackenzie has argued that the goal of the middle column was never descriptive when it came to finance. Rather, when markets did not conform to the tenets of mathematical economics, the *normative* quickly displaced the *positive* in the guise of financial engineering.

production decisions can often be represented as recipes of nested IF…THEN…ELSE statements, as in IF the inventories are below a threshold THEN ramp up production by 10%. The expressiveness/behavioral suppleness that is characteristic of *ACE* means that more realistic models can be built, ones that do not contain assumptions made merely for mathematical expediency. One may profitably consider using *ABM* in lieu of or in addition to mathematical analysis whenever relaxation of standard specifications produces analytical difficulties.

II *ABM* Antecedents and Exemplars

Like many technological innovations, *ABM* viewed as a recent invention is not the result of a single ground-breaking idea but rather a non-trivial recombination of many pre-existing technologies, in a way permitting a new kind of computational model. In this section we briefly describe key prior developments as they arose historically, to properly situate *ABM* within wider advances in computational science. While connections to physics, ecology, and evolutionary biology exist, the main touchstones for modern agent computing in economics and finance lie in long-standing connections to computer science, specifically *AI*, and *OR*.[8] We then go on to describe some *ABMs* that, early on, suggested the potential usefulness of the methodology for economics and finance.

A Pre-Modern History of *ABM* (1950s through the 1980s)

The notion of using digital computation to model individual households or firms and study the kinds of *aggregate* behavior produced at a higher, *social* level, grew up in the late 1950s among several distinct groups of economists (Orcutt, 1957, Clarkson and Simon, 1960, Shubik, 1960b). While their motivations were varied—some saw computation as a way around mathematical difficulties of aggregation, others thought the key advance was the ability to treat decision-making as dynamic—all were optimistic that digital computers would facilitate the creation of models having greater veracity than conventional ones, e.g., "…the simulation approach has emerged, as a practical means of studying and using more nearly realistic models of economic systems" (Orcutt, 1960).

The goal of one such approach was to move beyond Marshall's (1920) 'representative firm' and an 'average consumer' and build models having empirically-justified amounts of heterogeneity. This came to be known as *microsimulation* (Orcutt, 1957, 1960, Orcutt, Greenberger, Korbel and Rivlin, 1961) and is, in many ways, quite close in spirit to *ABM*. In microsimulation it is typical to model the behavior of many distinct households using more or less conventional constrained optimization, realized computationally (Bergmann, 1980, Bennett and Bergmann, 1986). Consumption and work decisions are represented (Bergmann, 1990), as are savings levels, tax compliance (Rohaly, Carasso and Saleem, 2005), and so on. Aspects of behavior may be specified probabilistically. Typically there are no networks, little learning or evolutionary behavior, a modest number of households, limited by available computer resources in early work, and while there is heterogeneity, it is largely exogenous. The models are run to equilibrium or a steady-state. While this approach has much in common with modern *ABM*, it is essentially different both conceptually and technically. Microsimulation took the neoclassical model of individual constrained optimization at face value, recognizing the importance of heterogeneity, and

[8] Mirowski (2001) provides an engaging historical analysis of the myriad interactions between these fields. However, he fails distinguish agent computing as something different. For agent-specific perspectives much closer to ours see Builder and Bankes (1991), Bankes (2002), Bonabeau (2002), or Axelrod (2003).

so the models that resulted were improvements over representative agent treatments and useful for policy but had little currency with theorists or others interested in richer behavioral specifications. Early researchers in this area would have benefitted greatly from today's programing languages and hardware, as is clear from Orcutt (1960), who laments that the 'building blocks' used in microsimulation models all had to be coded individually, while seeming to call for automated replication as is now common in all object-oriented languages. In essence, the pioneers of microsimulation had many of the same motivations as modern *ABM* researchers but inadequate tools, both too little hardware and too rudimentary software.

Contemporaneous with these developments, computational theories of the firm came into existence, led by the 'Carnegie School' of Herbert Simon, Richard Cyert and James March, all of the Graduate School of Industrial Administration (*GSIA*) at the Carnegie Institute of Technology in the late 1950s. Their research program aimed to better understand how firms actually behaved by studying a particular firm in great depth. Innovatively, they did this with the explicit goal of coding the activities of the firm in software, writing the model in *FORTRAN*. The resulting code served as the basis for the path-breaking book *A Behavioral Theory of the Firm* (Cyert and March, 1963), and opened up the study of individual organizations to computational approaches. The models in this book represented the behavior of individual employees, their roles and decisions, and so is an early example of a proto-*ABM*. Because these behaviors tended to be more rule following and heuristic than optimizing, it was different in character from microsimulation. Work in this tradition includes the later 'garbage can model of organizational choice' (Cohen, March and Olsen, 1972).

Wargaming and military simulation found widespread use in non-digital form in the first half of the 20th C. These tools and techniques were augmented by digital computers in the 1950s (Shubik, 1960a). They are a good example of qualitative, largely non-numerical computation in which no equations are solved. At this time same strategy games were adopted at business schools like the *GSIA* at Carnegie (Cohen et al., 1960), at UCLA (Jackson, 1959), and elsewhere (Cohen and Rhenman, 1961).

While the dominant use of early digital computing was for solving equations numerically, e.g., to support nuclear weapons research (Edwards, 1996, Mirowski, 2001) and the emerging field of weather forecasting (Edwards, 2010), other, non-numerical uses were both theorized and worked on, such as reproducing brain behavior (Ashby, 1952, 1956, von Neumann, 1958). Of interest as a precursor to *ABM* are *cellular automata* (*CA*)—simple finite automatons connected on a lattice—pioneered by Ulam (1952) and utilized by von Neumann for the creation of self-reproducing systems (von Neumann and Burks, 1966). This research investigated minimum specifications needed to have a machine reproduce itself logically. *CAs* became known to the scientific public in the 1970s through John Conway's *Game of Life* (Gardner, 1970).[9] Conway's two dimensional *CA* is particularly useful for illustrating ideas of *emergence* as it can produce a wide variety of persistent structures, including oscillatory *blinkers* and more complex ones that reproduce themselves spatially, such as *gliders*, all from very simple rules (see figure 2).[10] More directly relevant to economics, Peter Albin (1975) used *CAs* to model

[9] For more on the origins of the *Game of Life* see the recent biography of Conway by Roberts (2015).

[10] While not a model of any particular phenomenon, the *Game of Life* has many interesting properties (Bak, Chen and Creutz, 1989). By looking only at the rules it is very hard to see what will emerge (Faith, 1998).

economic development and others studied urban dynamics with them (Tobler, 1970, 1979, Couclelis, 1985, 1989). They played an important role in the rise of 'artificial life' (*ALife*) in the late 1980s (Langton, 1989, Langton, Taylor, Farmer and Rasmussen, 1992, Langton, 1994). *ALife* models typically feature some highly idealized representation of an ecological or social process using a population of simple computational entities in order to study phenomena at a higher organizational level (Hillebrand and Stender, 1994). Reynolds' (1987) demonstration that realistic-looking bird flocks could be generated from a few simple rules of individual behavior is a quintessential example.

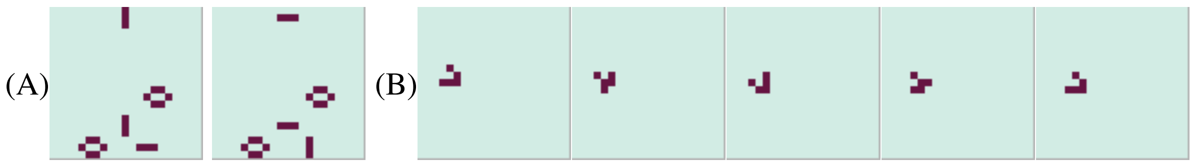

Figure 2*: The* Game of Life *unfolding on a 10x10 grid; each cell is either ON (red) or OFF (background) and has 8 neighbors (north, south, east, west, northeast, southeast, southwest, and northwest); all cells update in parallel according to the following rule: if the cell has exactly 2 neighbors ON its state is unchanged, if exactly three are ON it turns ON, otherwise it turns OFF;* (A) *shows two successive updates with 2 static patterns and 3 oscillating structures with period 2,* blinkers*;* (B) *shows a spatially-extended structure that repeats with period 4, a* glider*, moving to the southeast; images created with NetLogo*

We have suggested above that *ABMs* are a kind of non-numerical computing in the sense that there are typically no (or few) equations governing agent interactions that are 'solved' in the traditional sense. While individual agents can certainly use mathematical expressions in their own reasoning, macro (multi-agent) phenomena are produced in *ABMs* from individual actions that are aggregated in some simple, natural way, not according to some pre-specified equation that is 'solved' either directly or indirectly. This opens up *ABMs* to the possibility of *emergent* outcomes. These features—no equations being solved, individual actions that accrete to produce *emergent* social phenomena—are on display in perhaps the best-known *ABM* in economics, the late Thomas Schelling's model from the 1960s of residential segregation.[11] Schelling demonstrated that high levels of segregation could result from the bottom-up, decentralized home location decisions of individual households, none of whom had particularly segregationist preferences (Schelling, 1969a, 1971a, b, 1972b).[12] The idea that a modest number of whites moving away from small pockets of black immigration might 'tip' neighborhoods in major American cities is due to Grodzins (1958). Figure 3 shows the progressive segregation of a 100 x 100 spatial grid when agents are happy having as few as three of

[11] In Mirowski's (2001) otherwise admirable history of the *OR*-economics-computer science-game theory nexus he castigates the assertion that Schelling's model is the first true *ABM* as 'garbled history' (fn. 48, p. 369), revealing a limited understanding of the differences between the many kinds of simulation models that appeared early on and which have blossomed into completely different technologies today.

[12] Interestingly, Schelling's early efforts were *not* computational. Rather, he began working in one dimension but was convinced by Herbert Scarf that the exercise might be more clear in two dimensions (Schelling, 2006). Moving two kinds of coins around on a chessboard, with coins of the same kind 'preferring' to be next to one another, he showed that widespread segregation characterized long run configurations of the model. Later, while visiting the RAND Corporation, he had the model rendered computationally (Casti, 1994) but felt not much new was learned from this exercise. Given that modern computer displays had not been invented, such a model produced voluminous fanfold printouts, one for each time period, which certainly would make interpretation somewhat difficult. Later Schelling worked with a student to code a version of the model himself and wrote a long essay entitled "On Letting a Computer Help with the Work" (Schelling, 1972a) that was never published, but which makes clear he understood the value of computational renditions of his model (Hegselmann, 2012).

their immediate 8 neighbors like themselves in a population with equal numbers of reds and blues.[13] Unhappy agents move to any site where they would be happy.[14] Over time high levels of segregation emerge.

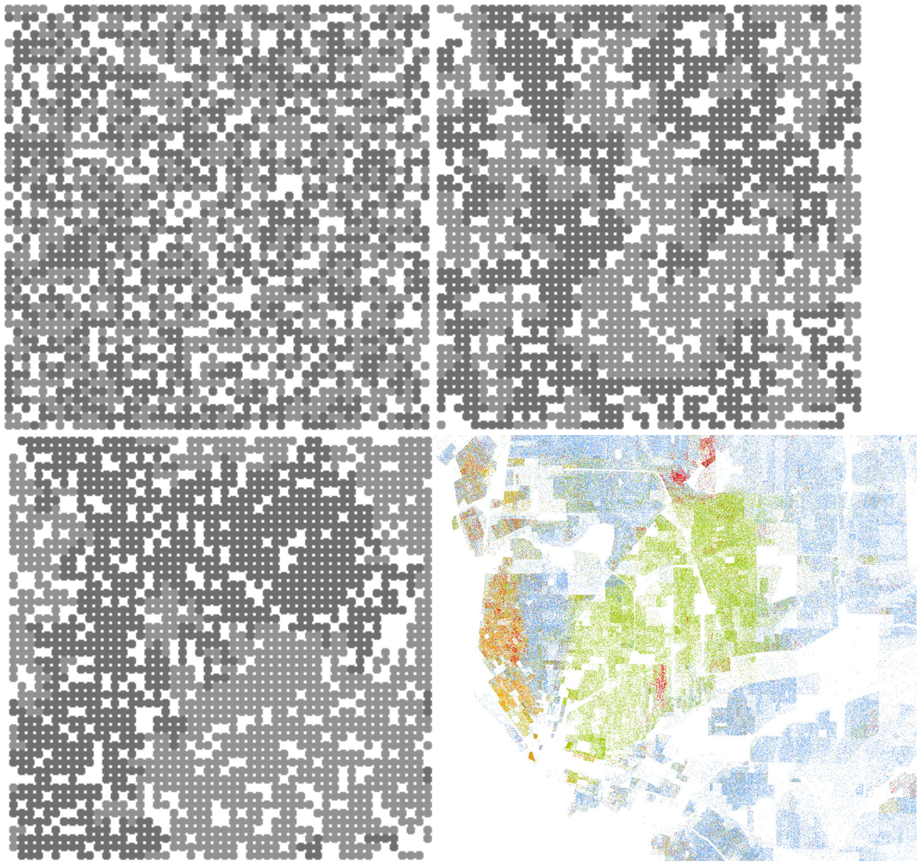

Figure 3*: Onset of segregation in Schelling model of residential choice from random initial conditions, and actual segregation in Buffalo, N.Y. c. 2010 (each person represented by one pixel)*

The panel on the lower right is a depiction of segregation in Buffalo, N.Y., data from the 2010 Census. Note the gross similarity. It is important to keep in mind that no agent prefers a highly segregated *aggregate* outcome to a more integrated configuration in the model, but in a world of distributed, decentralized action there are simply many more social configurations that look like the extreme outcomes on display in figure 3 than more integrationist ones, and so it is not clear how such outcomes might be prevented. Schelling's model has been elaborated in various ways, both to render it more realistic (Ingram, Kain and Ginn, 1972, Vandell and Harrison, 1978), to give it rigorous mathematical foundations (Young, 1998, Zhang, 2001, 2004a, b, Vinkovic and Kirman, 2006, Benard and Wiler, 2007, Pancs and Vriend, 2007, Dall'Asta, Castellano and Marsili, 2008, Gerhold, Glebsky, Schneider and Weiss, 2008, Zhang, 2011, Brandt, Immorlica, Kamath and Kleinberg, 2012, Barmpalias, Elwes and Lewis-Pye, 2014), to generalize it (Bruch and Mare, 2006, Benard and Wiler, 2007), and to bring it into the

[13] Schelling's model is sometimes called a *CA* but this is not strictly correct as in most version of it the agents are mobile *beyond* their local neighborhoods. *CAs* are usually defined by strictly local interactions.

[14] Technically, the ability of agents in the Schelling model to move anywhere on the grid means the model is not a traditional *CA*. Indeed, agents generalize *CAs*, permitting richer behavioral rules.

modern era of geographic information systems (*GIS*) (Crooks, 2010).[15] But its real power derives from its simplicity.

At about the same time Schelling was working on his model, Gordon Tullock and Colin Campbell (1970) were experimenting with simple models of voting and committee behavior in situations too complex to be solved analytically. They rendered their models computationally, as described at length by Wallick (2012), another proto-*ABM*.

By the 1970s the 'folk theorem' game theory, related to equilibria in repeated games, seemed to suggest that a wide variety of strategies could be supported as equilibria, providing little guidance for which strategies one might expect to encounter in real-world games. This situation served as the basis for an innovative computational experiment run by Robert Axelrod at the University of Michigan in the early 1980s. He solicited strategies to play the prisoner's dilemma game in round-robin fashion and received many submissions, from simple to complex. In the resulting tournament he discovered that a relatively simple strategy, submitted by Anatol Rappaport—so-called tit-for-tat—did very well against more elaborate strategies, effectively winning the tournament. He ran a second tournament with new strategies and found that tit-for-tat won again. He described his findings in a now classic book, *The Evolution of Cooperation* (Axelrod, 1984). While his tournaments consisted of a series of small-scale *ABMs*, each round involving only two agents interacting at a time, overall it represents an early use of heterogeneous computational agents in a competitive environment and was very novel at the time.

Of the several new computational approaches that came into existence in the post World War II period and were used by economists and the other social scientists—cellular automata, mathematical programming, microsimulation, digital war games, system dynamics, early agent-based computing, *AI*—each took a somewhat different approach to basic modeling abstractions, such as those in table 1. For example, the individuals in microsimulation are commonly households, in *CAs* a person is often associated with a lattice location and does not move, and in gaming the unit of analysis may be a strategic military resource like a plane or a ship, while *SD* models typically abstract from individuals altogether. All these modeling approaches feature rules for updating state variables. These can be interpreted behaviorally for *ABM*, microsimulation, and *CA*, but not for *SD*. The dynamics of *CA* always unfold on a network (albeit a lattice), which is often the case for *ABM*, but this is not common for microsimulation and is rare in *SD*. These and other features are compared in table 2.

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[15] Schelling's first descriptions of the segregation model were terse (Schelling, 1969a), unpublished (Schelling, 1969b), or somewhat informal (Schelling, 1971b), the latter appearing in a Washington-based policy journal that might more accurately be called a magazine of opinion than a publication with (social) scientific aspirations, i.e., one employing peer review systematically, etc. Meeting some resistance from economists, he described the model at length in volume 1, number 2 of the then new *Journal of Mathematical Sociology* (Schelling, 1971a). Remarkably, in the previous (very first) issue of that journal there appeared a paper having ambitions comparable to Schelling's, albeit considerably broader. It was entitled "The checkerboard model of social interaction" and authored by James Sakoda, a pioneer in computational sociology (Sakoda, 1971). The model developed by Sakoda admits Schelling's as a special case (Hegselmann, 2017), and derives from his dissertation (Sakoda, 1949). The latter is an extraordinary document, little cited but essentially a generation ahead of its time in seeing the possibilities of building computational models of a variety of social phenomena using *CA*-like specifications. Indeed, it is somewhat of a curiosity that Sakoda is so little cited, his *Journal of Mathematical Sociology* paper receiving 5-6% of the citations that Schelling's does, despite the fact that it appeared first!

Model feature	***CA***	**Microsimulation**	**Gaming**	***SD***	***ABM***
Individuals	yes	often households	yes	no	yes
Heterogeneity	states	states	states	no	states+rules
Behavior	fixed	optimizing	strategic	stock-flow	rule-based
Information	local	local+global	local+global	often global	local+global
Networks	always	never	sometimes	rarely	usually
Dynamics	internal	internal+external	internal+external	feedback	internal+external

Table 2*: Distinct approaches to computational modeling in the social sciences*

There is a definite sense in which *ABM* is the most general of these approaches, as it can be particularized to mimic any of the others. For instance, positioning agents on a lattice and giving them nearest neighbor interactions turns and *ABM* into a *CA*. Similarly, by using a large, homogeneous population of agents who behave based on the state of certain aggregate variables it is typically possible to represent *SD* models with *ABM*.

While each of these modeling approaches found a niche in the social sciences, developments in a few other fields permitted the *ABM* approach to come into its own in the early 1990s. We review these next.

B Advances in Computer Science Related to *ABMs*[16]

By the 1980s conventional *AI* had succeeded in building deep representations of highly restrictive domains (e.g., chess), but had largely failed to create anything like general-purpose intelligence. Distributed artificial intelligence (*DAI*) began from the perspective that agents can learn from one another (Gasser and Huhns, 1989). Very soon the individual *AIs* in *DAI* models were being given purposive behavior via utility functions, preferences, goals, and so on, and in a matter of a few years the field was transformed into multi-agent systems (*MAS*), with research monographs (Maes, 1990, Wooldridge and Jennings, 1995b, O'Hare and Jennings, 1996, Weiss, 1999) and textbooks (Ferber, 1999, d'Inverno and Luck, 2001, Liu, 2001, Wooldridge, 2002) soon appearing, including ones blending *MAS* concerns with game theory and economics (Parsons, Gmytrasiewicz and Wooldridge, 2002, Shoham and Layton-Brown, 2009).

All of this was happening as the Internet came to fruition and the idea of local, networked devices was replacing older notions of centralized computing facilities. At this time certain computer scientists began viewing groups of heterogeneous computing resources as *ecologies* (Huberman, 1987), invoking biological and economic formalisms for understanding how resources might be shared across devices, within networks. Curious ideas like market-based control (Clearwater, 1996) and market-oriented programming (Wellman, 1996) were put forward by crossing over ideas from economics and computer science, clearly indicating a certain appetite among computer scientists for importing extant ideas from the social sciences (Huberman and Hogg, 1994).

An important technical contribution of computer science to the agent modeling paradigm was object-oriented programming (*OOP*), which came of age with the *SmallTalk* and *Objective-C* languages in the early to mid-1980s. *OOP* refers to the encapsulation into software objects consisting of data elements (aka instance variables) and methods for manipulation of those data, resulting in self-contained and often reusable code. Programming with objects amounts to specifying typical patterns of object

[16] See Das (2016) for a recent survey of the literature in this field, with a particular focus on finance.

interaction, facilitating code reuse (Booch, 1994). It turns out that agents are quite naturally implemented as objects, and their interactions are succinctly captured by object methods. Today, *OOP* is standard for implementing *ABM* and *MAS* (Wooldridge, 2002).

C Developments in Operations Research Relevant to *ABMs*[17,18]

The idea of simulation is not new. Georges-Louis Leclerc, Comte de Buffon (1777) famously used an analog simulation to approximate π in the 18th Century (Grimmett and Strizaker, 1992). The widespread use of simulation coincides with the era of digital computation (Conway, Johnson and Maxwell, 1959, Conway, 1963). As digital simulation methods were adopted by the *OR* and related communities, there appeared several specialized languages for creating simulation models, including *GEMS* (from General Electric), *GPSS* (General Purpose Simulation System), *SimScript*—developed by future Nobelist Harry Markowitz—and *SIMULA*, the first object-oriented computer programming language (Banks and Carson III, 1984, Fishwick, 1995).

Transportation and traffic has always been an important subject within *OR*. Up until the mid 1990s it was conventional to model vehicular and pedestrian traffic as if it were fluid flow in a conduit (Transportation Research Board, 1961). Such models involved computational fluid dynamics, initially on mainframe computers and eventually on vector supercomputers. Later, researchers at Los Alamos National Lab (*LANL*) took an agent-based approach to the subject, giving driving rules to individual vehicles and studying traffic jams and related aggregate phenomena as *emergent* (Nagel and Rasmussen, 1994, Nagel and Paczuski, 1995, Nagel and Barrett, 1997). This proved to be a more flexible and useful approach, as it was possible to incorporate *GIS* map layers, road grids, and even real-time traffic data directly into models. Soon the *TRANSIMS* code (Barrett et al., 1995, Nagel, Beckman and Barrett, 1998) was being instantiated at city-scale, creating high-fidelity models of traffic flow involving millions of vehicles (Barrett and Beckman, 1995, Beckman, 1997). Indeed, traffic has been one of the great success stories of *ABMs* and today it is rare to find traffic models being built in anything other than an agent-based way. More recently pedestrian movement has been similarly revolutionized by *ABM* (Helbing, Farkas and Vicsek, 2000, Farkas, Helbing and Vicsek, 2002).[19]

Military operations were an early touchstone for *OR* techniques and *ABMs* have found wide use in this broad area. For example, force-on-force models have traditionally

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[17] See Macal (2016) for a recent overview of *ABMs* in management science and operations research and the book of North and Macal (2007) for applications to business.

[18] One set of developments in *OR* that is *not* very relevant to *ABM*, but which is sometimes confused with economic *ABMs,* concerns the computation of economic equilibria. Beginning with Scarf (1973, 1982) there is a large literature on efficient solution of fixed point problems in economics. This gave rise to certain specialized mathematical programming algorithms (e.g., the Eaves, Lemke, and Merrill algorithms (Todd, 1976)), in the wake of which so-called computable general equilibrium (*CGE*) models were born (Scarf and Shoven, 1984, Shoven and Whalley, 1992). Models of this type were widely adopted for policy and other applied work in economics and can yet be found in large institutional settings, e.g., the World Bank. Commercial software eventually emerged for the solution of *CGE* models (e.g., *GAMS*) and textbooks on the subject appeared (Thompson and Thore, 1992). Although such models can be used to represent the behavior of individual firms and other economic agents, they typically deal with only a few agents, or else represent an economy in overall supply and demand terms (i.e. no agents), solving for equilibria numerically. They are thus more 'top down' than 'bottom up' and do not focus on agent interactions. Such models are quite different from *ABMs* as the solution approaches are purely numerical and not representative of any kind of economic process. The same is true for the rather large literature on linear and nonlinear programming for solution of input-output models (Morgenstern and Thompson, 1976).

[19] Today, *ABM* is a key part of air traffic modernization (Calderón-Meza, 2011).

been treated as systems of differential equations, e.g., the Lanchester equations (Lanchester, 1916, Engel, 1954). Modern combat models are increasingly agent-based, in which each soldier, each weapon system, even each unit of ammunition present is treated as an object. Important early work on the *ABM* approach to combat models includes Ilachinski's *ISAAC* model and *EINSTEIN* software toolkit (2004).

D Agents in Ecology—*IBMs*—and in Other Branches of Biology[20]

Agent-based computational models in ecology began appearing in much the same way as in economics: sporadically before 1990 and more systematical after, driven by the same developments in computer science and operations research. In ecology it is conventional to call this 'individual-based modeling' (*IBM*) instead of 'agent-based modeling' or any of its variants.[21] Here the driving forces behind the development and adoption of an individual-based ecology (Grimm and Railsback, 2005) grew out of a basic dissatisfaction with population-based models. In mathematical ecology the basic models involve continuous populations with dynamics represented through ordinary and partial differential equations, often with space represented either explicitly or notionally. Formalisms in which individual animals are explicitly modeled are thought to have closer fidelity to the true heterogeneity and spatial variability present in the field (Fahse, Wissel and Grimm, 1998).

Two early and influential *IBMs* were the *JABOWA* model of forest dynamics (Botkin, Janak and Wallis, 1972) and a model of fish cohort growth due to DeAngelis, Cox and Coutant (1980). Grimm and Railsback claim that neither of these pioneering efforts saw *IBMs* as a general-purpose approach, i.e., as a paradigm. Rather, Huston, DeAngelis and Post (1988) articulated a more unified vision for the role of *IBMs* in ecology as Hogeweg and Hesper (1990) did later.

Progress in the 1980s and early 1990s in *CAs* (Wolfram, 1983, 1984, 1986, Toffoli and Margolus, 1987, Gutowitz, 1990, 1991) led to applications in biology (Ermentrout and Edelstein-Keshet, 1993), including neurobiology, developmental biology, population biology, even cancer oncology. Since then *CAs* have given way to *ABMs* in many areas (e.g., Schlesinger and Parisi, 2001, Zhang, Wang, Sagotsky and Deisboeck, 2009, Chapa et al., 2013, Norton and Popel, 2014, Wang, Butner, Cristini and Deisboeck, 2015).

Another area in which *ABMs* have had broad success is epidemiology, where traditional models based on ordinary and partial differential equations (e.g., Kermack and McKendrick, 1927) have been challenged and surpassed by agents, since the differential equation models essentially assume that populations are well-mixed (i.e., no network effects). In the wake of 9/11 the U.S. National Institutes of Health (*NIH*) created the *MIDAS* project (Models of Infectious Disease Agent Study) to catalyze the development of better models of infectious disease spread and response. *ABMs* were a primary focus of *MIDAS*. As a result of this project there now exist a large number of *ABMs* relevant to a

[20] A recent *ABM* textbook, while focused broadly, is written by ecologists (Railsback and Grimm, 2011).

[21] *IBM*, instead of *ABM*, was proposed as the name for the entire field back in the early days of the Santa Fe Institute when the SWARM agent modeling framework was first being created. It will come as no surprise that when 'individual-based modeling' was proposed by some as a good descriptor it was vetoed by Chris Langton since the acronym for 'individual-based modeling' is identical to the standard name of a large U.S. corporation active in the computing field, which was thought to be at least confusing if not more sinister. Another name for the field that did not stick was 'actor-based modeling' for in sociology so-called 'stochastic actor theory' exists as an active research program, focusing mostly on social network models.

variety of diseases—influenza, smallpox, *SARS*, *MERS*, Ebola, West Nile virus, Zika—at both the national and international levels (e.g., Halloran, Longini Jr., Nizam and Yang, 2002, Eubank et al., 2004, Longini Jr. et al., 2005, Carley et al., 2006, Gemann, Kadau, Longini Jr. and Macken, 2006). Epidemic *ABMs* have also been applied to a variety of pathological behaviors that have social origins, including smoking (Wallace, Geller and Ogawa, 2015), drug addiction (Agar and Wilson, 2002, Hoffer, Bobashev and Morris, 2009, Heard, Bobashev and Morris, 2014), and obesity (Hammond, 2009).

E *ABM* in Economics and Finance

In this section we look at the literature on agents in economics and finance from the past 25 years, describing some of the most influential work. An early statement of the potential of *ABMs* is due to Holland and Miller (1991). For a view of *ABMs* as 'artificial economies' see Lane (1993a, b). The *Handbook* edited by Tesfatsion and Judd (2006) is a good source up through 2005 or so—we distill the best from it, mentioning many older models that have become *ABM* classics, and move on to important recent work.

1 Microeconomics and markets

Arthur introduced the so-called El Farol or 'bar attendance' problem (Arthur, 1991, 1994) as a paradigm for inductive learning in contrast to rational behavior. In his model there is a population of agents all of whom have the same preference for attending a club that evening. If the club is either too crowded or too empty it is no fun for any of the attendees. Arthur demonstrated that with enough heterogeneity in forecast functions the population can 'grow' good solutions so that the bar has very nearly the right number of people attending each week. It is a paradigm for heterogeneous agents arriving at mixed strategy Nash equilibrium despite none of them trying to compute such a thing. This model has generated a large secondary literature (Bell and Sethares, 2001, Bell, Sethares and Bucklew, 2003) and in finance has come to be known as the 'minority game' (e.g., Jefferies, Hart and Johnson, 2001, Johnson, Jefferies and Hui, 2003).

A wide variety of *ABMs* have been built to study markets conceived in the Walrasian sense but without a centralized auctioneer (Leijonhufvud, 1967).[22] This work includes Albin and Foley (1992, 1998), who looked at distributed, decentralized bilateral trading with local price formation and contrasted their results with Walrasian outcomes, focusing on the effects of price dispersion when no auctioneer is present, including welfare effects associated with the production of *horizontal inequality*.[23] The *Sugarscape* model of Epstein and Axtell (1996) extended Albin and Foley with heterogeneous agents, endogenous interactions, changing preferences, and so on. Wilhite (2001) investigated this same class of models for agents connected in various network topologies. Vriend (1995) looked at the self-organization of markets using a classifier system and later asked how such models relate both to the 'invisible hand' (Kochugovindan and Vriend, 1998) as well as to Austrian market process theory (Vriend, 2002). Axtell (2005) studied the

[22] Analytical models of this type include Rader (1968), Feldman (1973) and Bell (1997), who derive conditions under which decentralized exchange yields Pareto optimal allocations, results analogous to the conventional welfare theorems of general equilibrium.

[23] This refers to differences in welfare that arise between twins—agents having the same preferences and endowments in pure exchange—and was first investigated by Foley (1994). Horizontal inequality cannot arise in standard Walrasian equilibria because of the so-called *equal treatment property* (Green, 1972).

computational complexity of such markets and proved that exchange at local prices yields allocations in polynomial time—linearly in the number of agents and quadratically in the number of commodities—in contrast to well-known computational hardness results from computer science for computing Brouwer fixed points (Papadimitriou, 1994). In essence, such decentralized exchange processes act like a giant distributed computation of Pareto optimal allocations with final prices representing the marginal rates of substitution that all agents converge to in equilibrium.[24] All of the above results pertain to pure exchange economies. Gintis built an *ABM* economy with production and studied its convergence to general equilibrium, finding many of the same phenomena on display in pure exchange, e.g., initial price dispersion, welfare effects, and so on (Gintis, 2007).

Kirman and Vriend (2000, 2001) took a more empirical approach in a model of the Marseille fish market, where they discovered that buyer-seller loyalty plays a large role and is often more important than price in purchase decisions. Vriend (2004) reviews a variety of distinct market forms that have been studied with *ABMs*.

Continuous double auctions (*CDAs*) have been investigated using a tournament approach (Friedman and Rust, 1993, 1994), analogous to Axelrod's prisoner's dilemma tournament, with individual participants submitting computational strategies. No single strategy seems to always do well in such environments, while many ostensibly sophisticated strategies seem to fare poorly. *CDAs* are commonly used in real-world markets, for electric power, Treasury securities, and so on, and *ABMs* of such markets have appeared (e.g., Nicolaisen, Petrov and Tesfatsion, 2000, Koesrindartoto, 2004, respectively). Other kinds of auctions have been studied positively with *ABM*s (e.g., Hailu and Schilizzi, 2004, 2005, Hailu and Thoyer, 2006, Hailu and Thoyer, 2007) while computer scientists have provided software tools for configuring a huge variety of auction types (Wurman, Wellman and Walsh, 1998).

Markets with information asymmetries, like those proposed by Akerlof (1970) and Rothschild and Stiglitz (1976), have been investigated with *ABMs*, including Tilles et al. (2011) and Comer (2017). One finding in this literature is that the way asymmetries are implemented seems to make some difference in the kinds of results produced.

Coalition formation has been studied by both economists and computer scientists using agents. The former tend to be more concerned with constraints on the generation of realistic-looking groups (De Vany, 1993a, b, 1996a, b, c) while the latter are often concerned with the complexity of producing groups (Shehory and Kraus, 1993, Klusch and Shehory, 1996a, b, Sandholm et al., 1998). The number of possible coalitions for any realistically sized population is so vast that models of coalition evolution are not plausible as mechanisms for the creation of anything like optimal groups of agents.

Another microeconomic topic studied with *ABM* is bipartite matching, which to has a wide variety of real-world applications, including marriage, college admissions, assignment of medical residents, and so on. The well-known Gale-Shapley (1962) algorithm is known to produce stable solutions via the so-called 'deferred acceptance' mechanism, in a wide variety of circumstances (Roth and Sotomayor, 1990). Unfortunately, it yields extremes of welfare, with proposers get maximal payoffs while acceptors receive the minimum (Knuth, 1976). Gale-Shapley matching is a centralized mechanism as no pair can be considered finally matched until all agents are paired. This

[24] While Cheng and Wellman (1998) have shown how to compute Walrasian equilibria in distributed fashion, their algorithm does not represent any economic process that takes place in real economies.

is highly unrealistic descriptively (e.g., of marriages or colleges) and has led to the search for decentralized matching models that have 'nice' properties. Henrickson (2002) and Fuku, Namatame and Kaizoji (2006) is one attempt to produce a decentralized solution as is Axtell and Kimbrough (2008). The latter find that distributed mechanisms exist having a very small number of unstable pairs. The existence of such pairs is viewed as unlikely to lead to unraveling due to the low probability that they encounter one another.

2 Finance

One of the most active areas of *ABM* has been in finance. From agent-based stock markets featuring software traders to banking regulation and financial crisis modeling, a large and growing literature of *ABMs* has grown up dealing with finance topics.[25]

One of the very first modern agent-based models was the Santa Fe Stock Market (Palmer, Arthur, Holland, LeBaron and Tayler, 1994, Arthur et al., 1997, LeBaron, Arthur and Palmer, 1999), an artificial financial market featuring traders who coevolved their behavior in response to market conditions. Agents traded one risky and one riskless asset, with learning represented by a Holland-type classifier system. This artificial market has rich behavior, including both a rational expectations equilibrium in which essentially no trading occurs, and a more realistic regime in which there is extensive trading characterized *qualitatively* by clustered volatility and a few other stylized phenomena from real financial markets. This *first generation* artificial stock market did not have strong empirical ambition, the main goal being to create a credible model of a financial market in software. Similar efforts followed on this work, often with different behavioral mechanisms, as in the use by traders of genetic programming to predict future prices (Chen and Yeh, 1997).

There followed a group of relatively simple models with deeper empirical ambitions, capable of reproducing many of the *quantitative* facts about markets, e.g., heavy-tailed return distributions, clustered volatility, no auto-correlation of returns, but significant auto-correlation in absolute returns (Cont, 2001). These models include Lux and co-authors (1998, 1999, 2000), Levy, Levy and Solomon (2000), LeBaron (2001a, b, c, d, 2002), Farmer and Joshi (2002) and Cont and co-workers (Ghoulmie, Cont and Nadal, 2005, Cont, 2006). Typical in these models are modestly-sized populations of traders—typically in the 100s—with sub-populations of informed and uninformed traders, the relative fractions of each being endogenous, depending on market conditions. Efforts with this *second generation* of artificial stock markets clearly demonstrated that many empirical facts could be reproduced with highly parsimonious agent models. Sometimes innocuous features of these models turned out to be play important roles, as when it was demonstrated that as the number of agents in the Lux model increased various measures of volatility decayed monotonically such that in the limit of infinite agents there were no dynamics at all (Egenter, Lux and Stauffer, 1999).

With the better part of a decade's worth of artificial financial market modeling experience, it was becoming clear at the turn of the Century that there were other ways to think about and understand financial markets than in the usual equilibrium market microstructure way (O'Hara, 1997). For instance, Farmer's view of such markets as an ecology of interacting agents, each with local, idiosyncratic forecasting functions, became a viable alternative (Farmer, 2002). According to this perspective, non-

[25] For an excellent overview of this literature see LeBaron (2006).

equilibrium price formation can lead to oscillatory price dynamics, value investing may not lead to prices following values, trend following may generate short-term trends, there can be boom-and-bust cycles, and so on, a much richer and potentially more realistic view of financial markets than the conventional one. The rightmost column in our table 1 above also holds many desiderata for models in finance, and *ABMs* are one way to start the trek from the middle column rightward.

ABMs of more specialized markets also appeared, such as those of Arifovic and co-workers who modeled foreign exchange markets using agents who learn via genetic algorithms (Arifovic, 1996, 2001). Specific financial market phenomena have been modeled with *ABM*s including so-called 'flash crash' events (Paddrik et al., 2012). High-frequency trading (Leal, Napoletano, Roventini and Fagiolo, 2016) and transaction taxes (Fricke and Lux, 2015) have also been studied with this versatile methodology.

In parallel with these *pure ABM* models there arose a literature involving a small number of fundamental and technical traders, models that had the advantage of being analytical tractable to some extent (Brock and Hommes, 1998). In this line of inquiry the kinds of phenomena the models can produce include highly nonlinear dynamics and chaos (Hommes, 2002). Attempts to validate these simple, abstract models with laboratory experiments have been made (Hommes, 2011).

These early successes with financial market *ABMs* led to their practical use in various ways. At the end of the last Century the Securities and Exchange Commission (*SEC*) ordered the *NASDAQ* market to move from trading in terms of eighths and sixteenths of a dollar to pennies—decimalization. The *NASDAQ* had just been hit by lawsuits concerning so-called 'spread clustering' and its management was keen not to jeopardize market performance with regulatory changes. It therefore decided to build a high-fidelity *ABM* of its market, using the proprietary data only it had access to, including many institutional details that had not previously been built into agent-based financial markets. For instance, the so-called 'small order execution system' (*SOES*) was a set of protocols for dealing with certain kinds of orders, and this was to be modeled. Over the better part of a year an *ABM* of the *NASDAQ* was created and its performance was calibrated to actual market behavior. (Darley, Outkin, Plate and Gao, 2001). It was an evolutionary model and made several predictions as to what the effect of decimalization would be on overall market function. These were made in advance and after the market rules changes most of the predictions were born out (Darley and Outkin, 2007). This use of *ABM* to address the effects of alternative regulations has since been duplicated in evaluations of alternative 'circuit breakers' for financial markets (Yeh and Yang, 2010).

In the last decade there has appeared what might be considered the *third generation* of financial market models, involving some representation of order books, that is, the set of orders poised to execute depending on price movement. These models typically involve some order book data from real markets, necessary for calibration or estimation of model parameters, and includes Farmer and co-authors (2005, 2008), Preis, Golke, Paul and Schneider (2007), Feng et al. (2012), among many others.

All of the financial market models above involve one risky asset. In recent years models with multiple risky assets have been created, involving agents to buy portfolios, making the models significantly more complex (Yang, Wang, Sun and Wang, 2015). Social network influence has also been added to artificial stock markets. It remains to be seen if these additions produce new phenomena, thus warranting the title *fourth generation* models. Table 3 compares the different kinds of financial market *ABM*s.

Generation	***Empirical orientation***	***Market-clearing***	***Risky assets***
First	qualitative	each period	1
Second	quantitative	each period	1
Third	quantitative	order book	1
Fourth?	quantitative	order book	many?

Table 3*: Evolution of artificial financial market models*

Agent models of bank behavior, including intra-bank decision-making about risk and interbank networks have appeared (Nier, Yang, Yorulmazer and Alentorn, 2008, Minoiu and Reyes, 2011, Minoiu, Kang, Subrahmanian and Berea, 2013, Caccioli, Shrestha, Moore and Farmer, 2014), mostly since the Great Recession. Innovative techniques for studying leverage and stress tests have grown up either in *ABM* form or with agent inspiration (Thurner, Farmer and Geanakoplos, 2012, Poledna, Thurner and Farmer, 2014, Aymanns and Farmer, 2015, Klimek, Poledna, Farmer and Thurner, 2015).

Housing models using agents have begun to appear (e.g., Gilbert, Hawksworth and Swinney, 2009). Together with Peter Howitt, John Geanakoplos, and a number of our students we have built such a model for the housing market bubble in the Washington, D.C. metro area c. 2000-2010 (Geanakoplos et al., 2012). This model uses administratively complete data on housing stock from county records, data on mortgages from CoreLogic, data on households from various sources, and attempts to match the universe of real estate transactions during this period, data acquired from the local *MLS*. This *ABM* is somewhat unusual insofar as it can be run, in principle, at 1-to-1 scale with the actual regional economy under study. Giving each household intelligible rules of behavior for home-buying and selling, applying for a mortgage, paying taxes, refinancing, and so on, we have found that it is possible to closely calibrate the model to the actual events, matching a variety of time series both qualitatively and quantitatively, as shown in figure 4. In addition to the overall price bubble, we have been able to do a good job on the inventories of homes for sale, original listing versus actual sale price, days-on-market, loan-to-value of new mortgages, and so on. All of these quantities changed structurally over the course of the bubble, with inventories and days-on-market shrinking during the price run-up and then greatly expanding as the bubble burst. There are other aggregate variables that our model was not able to reproduce closely, such as the overall home ownership rate. This was almost surely due to having very limited data on the rental market. The work on housing was undertaken as a first step toward better understand the Financial Crisis of 2007-9.

Calls for using agent modeling to both understand the Crisis overall and to avoid similar events in the future have appeared (Buchanan, 2009, Farmer and Foley, 2009, Battiston et al., 2016). Since then several *ABMs* have appeared. Because most of these models have important macroeconomic submodels we defer discussion of them to §6 below. There are a few agent models that deal with particular parts of the financial system and their behavior leading up to and during the of Crisis, such as the role of mortgage securitization (Goldstein, 2017). *ABMs* seem well-suited for this kind of effort, although such work is in its infancy today. Agent models of other types of financial crises, e.g., currency crises, have also appeared (Arifovic and Masson, 2004).

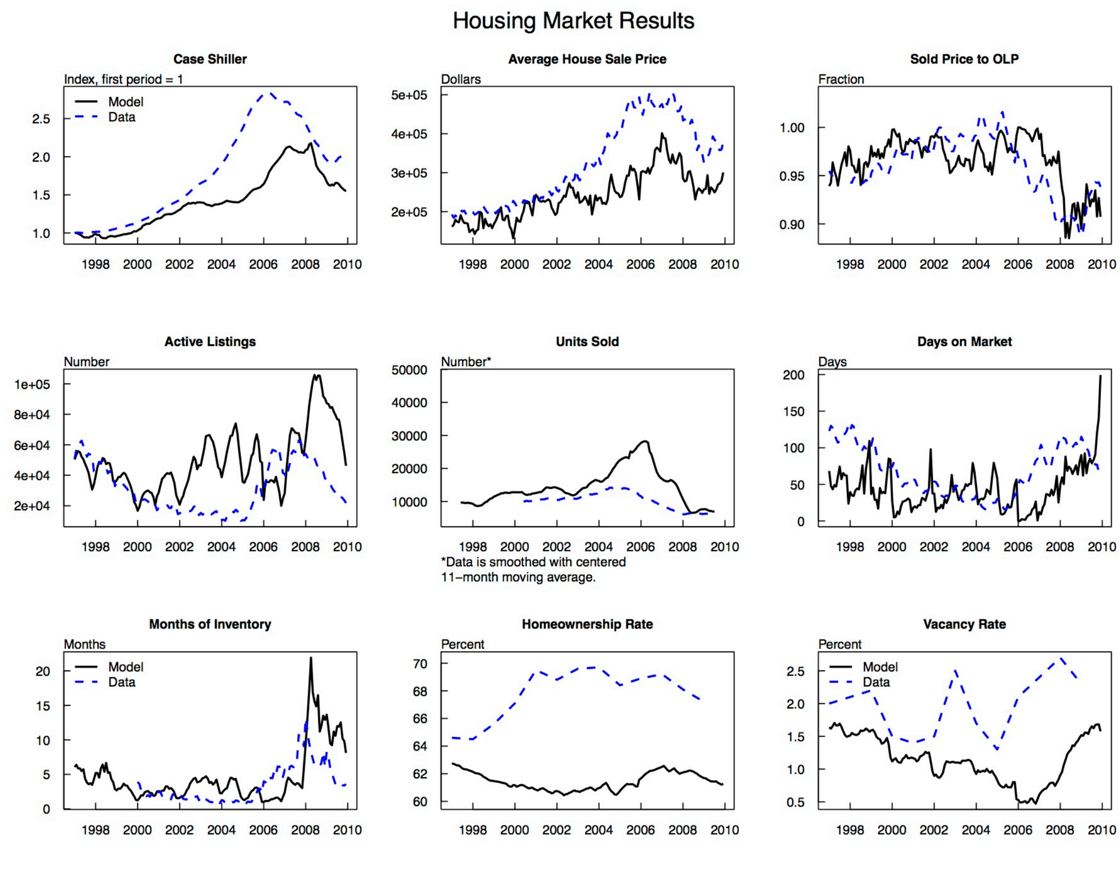

Figure 4: *Output from an agent-based housing market bubble model, with data shown as dashed lines and model output solid black lines*

3 Game theory

We have mentioned the game theoretic tournament run by Axelrod in the 1980s, as proto-*ABM*. It turns out that some of the early work in *Alife* had strong game theoretic orientations as well, including Lindgren's evolutionary solution to repeated play of the prisoner's dilemma (Lindgren, 1992), in which an arms race erupts between players and ever more complex strategies progressively dominate simpler ones; see also (Bankes, 1994). A small literature on finite automata players in game theoretic settings grew up at about this same time (Rubinstein, 1986, Miller, 1989, Binmore and Samuelson, 1992, Rubinstein, 1998). Most of these efforts were analytical, not computational.

At about this same time, evolutionary biologists began using computational agents more clearly related to modern *ABM* in order to study spatial games. Nowak and May (1992) fixed the position of simple agents in a two-dimensional space and let them play the prisoner's dilemma with their neighbors, using parallel updating, in essence a *CA*. The discovered relatively high levels of cooperation in beautiful, dynamic, transient patterns, their paper in *Nature* essentially screen captures from their model, this despite the fact that 'defect' is a dominant strategy! Soon thereafter (Huberman and Glance, 1993) demonstrated that the Nowak and May results were artifacts of the synchronous updating mechanism, and that any relaxation of it whatsoever—any amount of asynchrony—broke all the beautiful patterns and caused the cooperative results to unravel into pure defection, the expected result.

An important motivation for game theoretic *ABMs* has been to relax rationality and other conventional specifications (Moss, 2001a), in the spirit of table 1 above. There are *ABM*s using game theory set-ups that add networks, k-level cognition, and so on.

The emergence of social norms, conventions and institutions is often formulated in game theoretic terms, since the main question is whether individuals have incentive to comply with them. This topic that has received considerable attention from economists (Kandori, Mailath and Rob, 1993, Young, 1998, Burke, Fournier and Prasad, 2006) and other social scientists (Coleman, 1964), as well as from philosophers (Lewis, 1969, Bicchieri, Jeffrey and Skyrms, 1997, Bicchieri, 2006) and computer scientists (Rosenschein and Zlotkin, 1994, Walker and Wooldridge, 1995, Shoham and Tennenholtz, 1997, Ossowski, 1999). There are a large number of *ABMs* of norms and conventions that have appeared (Axtell and Epstein, 1999, Axtell, Epstein and Young, 2001, Epstein, 2001, Hales, 2002, Eisenbroich and Gilbert, 2014).

For *MAS* researchers using strategic agents and therefore more or less conventional game theoretic settings, much of what passes for research today is applied rational choice game theory (Shoham and Layton-Brown, 2009), little related to *ABM*.[26]

4 Industrial organization, firms, and organizational behavior

An early *ABM* applied to industrial organization was Marks' use of the genetic algorithm to 'breed' better oligopolist strategies (Marks, 1992). Mazzucato later used replicator dynamics, a close 'cousin' of *ABM*, to model competition between firms (Mazzucato, 1998). Kimbrough and co-authors have attempted to reproduce most of conventional rational choice oligopoly theory—Cournot and Bertrand competition, etc.—using simple agents via *ABM* (Kimbrough and Murphy, 2009, Haas, Kimbrough and van Dinther, 2013, Kimbrough and Murphy, 2013). These agents do not have deep internal models of how their local economies work, rather they 'probe' their economic environment for performance gradients and 'adjust' their behavior accordingly, moving in the direction of higher profits. Perhaps surprisingly, these simple models do a good job reproducing most of the content of the rational theory.

Luna investigated the formation of firms (Luna, 2000). Axtell built a model in which firms 'grow' from the decisions by people to start new firms or leave old firms (Axtell, 1999). It can be calibrated to produce a variety of outputs that closely resembles empirical data on U.S. firms (Axtell, 2002). A high-fidelity *ABM* of Japanese firms tied closely to data has been built (Aoyama et al., 2010).

Agent models of firm operations is a very active area of research, including work on supply chains (e.g., Lee, Padmanaabhan and Whang, 1997), marketing (e.g., Rand and Rust, 2011), customer behavior (e.g., Said, Bouron and Drogoul, 2002), diffusion (e.g., Garcia, 2005), e-commerce (e.g., Glushko, Tenenbaum and Meltzer, 1999), manufacturing (e.g., Leitao, 2009), and so on. A significant fraction of this literature spills over into management science and is too large to be succinctly summarized here. Among well-known *ABMs* is the supply chain model of (Parunak, Savit and Riolo, 1998), notable for contrasting *ABM* results with a more conventional equation-oriented model. The enormous literature describing product diffusion (Rogers, 1995, Valente, 1995,

[26] An emerging field in computer science known as *algorithmic game theory* (Nisan, Roughgarden, Tardos and Vazirani, 2007) focuses on computational issues. Similarly, *computational social choice theory* has grown up as a field within computer science (Brandt et al., 2016). Neither of these areas falls within *ABM*.

1996) has long had a 'bottom up' perspective while corresponding mathematical formalizations tend to be more aggregate in character (Bass, 1969). There is a rapidly growing *ABM* presence in this area that has been recently reviewed (Kiesling, Günther, Stummer and Wakolbinger, 2012). Rahmandad and Sterman (2008) contrast agent models of diffusion with mathematical approaches. Much of the work on diffusion is closely related to models of opinion dynamics (e.g., Goldenberg, Libai and Muller, 2001, Deffaunt, Amblard, Weisbuch and Faure, 2002, Hegselman and Krause, 2002).

Organization theory is a very active area for *ABM*. An early statement of this research program is Carley and Prietula (1994). Computational organization theory is increasingly agent-based, as evidenced by the edited volumes of Prietula, Carley and Gasser (1998) and Lomi and Larsen (2001). A good but somewhat dated overview is Carley (2002). A wide range of topics is dealt with in the literature, including information flow within organizations, hierarchy and power relations, compensation, work effort, and monitoring issues, learning curves, efficiency, the trade-off between exploration and exploitation, and so on. For example, worker turnover has been investigated with *ABM* by Dal Forno and Merlone (2004) while Phelan (2004) has studied promotion policies.

5 Labor economics

There has been considerable work studying labor markets using *ABM*, including Tesfatsion (1998, 2001, 2002, 2003), Fagiolo, Dosi and Gabriele (2004), Richiardi (2004), Neugart (2004), 2008) and others. The typical motivation for these studies is either to 'grow' aggregate labor market performance measures, like Beveridge curves and matching functions, form the bottom up, or to relax one or more of the conventional assumptions in standard labor economics, such as homogeneous workers, uniform reservation wages, rational decision-making, and so on. Notable about several of these studies is the explicit focus on policy issues, such as the size and duration of unemployment payments, and so on. The expressiveness of *ABM* is helpful here.

The role of social networks for job referral has been studied for some time (Granovetter, 1973, 1995). The effect of such networks on economic outcomes, via segregation, the production of inequality, and so on, has been investigated both analytically (Calvó-Armengol and Jackson, 2004, 2007, 2009) and with *ABM* (Tassier and Menczer, 2001, 2008). While idealized networks may facilitate analytical solutions, realistic networks usually means turning to *ABM* (Jackson, 2008).

Recently, longitudinal employer-employee matched data for whole countries have become available. These data permit the construction of networks between firms formed by workers who follow employment opportunities from firm-to-firm. These have been termed *labor flow networks* (*LFN*) since they describe how labor moves between firms. *ABM*s have been built to reproduce these networks (Guerrero and Axtell, 2013). Conceiving of employment relations through *LFNs* brings new ideas to labor economics, like firm-specific unemployment and network-induced excess unemployment, breathing new life into old ideas like the spatial mismatch hypothesis (Wilson, 1978).

6 Macroeconomics and policy

An early macro model with agents is Bruun (1999), which has a Keynesian structure. Since that time there have appeared a number of *ABMs* that attempt to move beyond the representative agent, to include more agents and to make them heterogeneous.

Aoki (2001) showed that interactions between agents could provide new insights for macro. Allen and Carroll (2001) studied consumption behavior in a population of imitators. Bullard and Duffy (2001) modeled agents learning about macro volatility. Many people have offered visions for what *ABM* macro might look like without providing working models (Axtell, 2006, Tesfatsion, 2006, LeBaron and Tesfatsion, 2008). Recent efforts focus on heterogeneous agents and social learning (Palmer, 2015).

Howitt and Clower (2000) studied the emergence of money in a model that featured many goods and stores selling those goods, with barter arrangements. An earlier *ABM* on the same topic is Marimon, McGrattan and Sargent (1990).

About a decade ago there arose the idea of bringing high-performance computing to bear on macro using large-scale agent models. The so-called EurACE model (Cincotti, Raberto and Teglio, 2010) was the first example of this endeavor. It yielded a model featuring some 10 thousand consumers and firms, generating a variety of macroeconomic phenomena. Eventually the model was made policy relevant and can today be exercised to study policy alternatives (Dawid et al., 2012).

At about the same time, and as the Financial Crisis began to unfold, ideas of 'emergent macro' and 'macro from the bottom up' were invoked to study 'financial fragility'. These models featured populations of consumers, employed in firms, who borrow from banks to operate. The linkages between firms and banks can lead to credit crises and recessions. This literature includes Delli Gatti (2008, 2011).

Conventional *DSGE* models do not feature a financial sector. Given the success of *ABM* financial markets, the so-called *CRISIS* project proposed blending agent-based finance and macro models in order to study the events surrounding the recent global crisis—how it unfolded and how to ameliorate its effects. This effort combined macro and financial sector models in order to produce credible bottom up dynamics of lenders (banks), households, investors, regulators, and consumers. It produced several interesting *ABMs*, including high quite detailed models of banks, bankruptcy resolution, and so on.

7 Environmental economics

Many problems in natural resource and environmental economics involve features from the right column of table 1, such as spatial processes, networks, and disequilibrium dynamics. *ABM* has been widely applied in this area of economics.

Common pool resource management was brought to prominence by Ostrom (Ostrom, 1990, 1994, 1999), who advocated *ABM* methods late in her life. The ability of groups of people to manage their own exploitation of scarce resources begs for realistic models. For renewable resources *ABM* has been used for some time (Antona et al., 1998, Rouchier et al., 2001). For fisheries a variety of *ABMs* have appeared (Bousquet, Cambier and Morand, 1994, Bousquet, 1996). Overviews of this literature include Bousquet and Le Page (2004) and Janssen (2002).

Many of the problems that afflict macroeconomic models—high degree of aggregation, focus on representative agents and equilibrium—manifest themselves in models climate change economics (Nordhaus, 1993b, a). In response a number of agent-based models relevant to climate change have appeared (Downing, Moss and Pahl-Wostl, 2001, Moss, Pahl-Wostl and Downing, 2001) (Gerst et al., 2013). What is still missing from these is a true bottom up perspective, the case for which is made by Farmer, Hepburn, Mealy and Teytelboym (2015).

Land use and cover change are important topics in agricultural economics and related areas. These have proven fertile ground for *ABM* because they demand the representation of spatial processes. This is a large literature and has been summarized by Parker et al. (2003) and more recently by Matthews et al. (2007). A closely related topic is markets for land. Because the value of land depends importantly on its spatial proximity to other economic goods and services, spatial models are again a key motivation for *ABM*s. There is a large and growing literature on this topic (Filatova, Parker and van der Veen, 2007, Filatova, 2009, Filatova, Parker and van der Veen, 2009, Magliocca, Safirova, McConnell and Walls, 2011, Filatova, Verburg, Parker and Stannard, 2013). The usefulness of agents for agricultural economics was pointed out by Berger (2001) who also suggested that *ABM* policy analysis was readily accomplished.

An interesting variant of *ABM* that has grown up largely within environmental applications is known as *participatory modeling*. In keeping with the bottom up spirit, when stakeholders can be engaged in the modeling process they can be given a role in the model and are then permitted to act in lieu of the artificial agent. This approach has found success in a variety of natural resource environments (Siebenhüner and Barth, 2005).

8 Public economics, political economy and public policy

Agent computing has been used to study a wide variety of topics in political economy, broadly construed, from the transition to agriculture (Bowles and Choi, 2013) to taxation (Mittone and Patelli, 2000), public choice (Wallick, 2012), and regional economic issues (Zhang, 2003).

As mentioned at the start of this review, models of taxation have long been the province of microsimulation, insofar as that computational methodology permitted incorporation of detailed data on taxpayers into analyses. This ability proved both a strength and a weakness. Clearly, detailed micro-data are necessary if accurate assessments of changes in the tax code are to be made. However, this extreme detail, combined with the fact that households do not interact in conventional microsimulation, means that a very large number of parameters are needed to march those models forward in time, data that often does not exist. Therefore, there has been a slow but steady migration of tax models to *ABM*, both as a way to study interactive behaviors not easily studied with microsimulatiton, but also because many of the life behaviors that require parameters are more naturally represented in agent models (e.g., events like marriage or divorce). It is also the case that agent models facilitate the representation of boundedly rational taxpayer behavior, important in models of compliance (Antunes, Balsa, Respicio and Coelho, 2007, Korobow, Johnson and Axtell, 2007, Bloomquist, 2010, Hokamp and Pickhardt, 2010). Models featuring social networks of taxpayers, and through which various kinds of information flows, are naturally studies with agents (Bloomquist, 2012, Andrei, Comer and Koehler, 2014).

Several researchers have looked at the *ABM* approach, generally, and have pointed out that its deep expressiveness combined with the abilities to interact with stakeholders (see section 7 immediately above) and communicate to decision-makers combine to create a methodology that is potentially very useful for policy (Lempert, 2002, Moss, 2002, Gulden, 2004). Against this perspective, Durlauf has argued that the complexity approach to economics, overall, and *ABM* methodology, specifically, "does not fundamentally affect policy evaluation" (Durlauf, 2012: 68).

F *ABM* in Politics, Sociology, Demography, Geography, and Anthropology

There has been systematic use of *ABM* in *quantitative* branches of other social sciences. An overview of the span of agent computing across the social sciences is Berry, Kiel and Elliott (2002), although somewhat dated now. *ABM* methodology is an integral part of the emerging field of computational social science (Chen, Terano, Yamamoto and Tai, 2014), along with data-intensive methodologies like machine learning (Lazer et al., 2009, Watts, 2013). Here we review important *ABM* work in several disciplines, with opportunity to be brief due to good disciplinary-specific surveys appearing recently.

Within political science early use of *CA* includes models of international relations (Cusack and Stoll, 1990). Work of Axelrod is very much in the *ABM* tradition (Axelrod, 1995, 1997a) while that of his student Cederman has more of a *CA* flavor insofar as it describes the behavior of countries on landscapes (Cederman, 1997, 2001b, 2002, 2003). A dated but useful overview of *ABM* in political science is Cederman (2001a). A more recent but less *ABM* focused overview is de Marchi (2005).

There has been a variety of work on voting systems, redistricting, gerrymandering, and so on using *ABM*. The Tiebout model purports to describe residential location decisions based on local public goods (Tiebout, 1956), and can produce sorting although the empirical status of such sorting in the U.S. is disputed (Dowding, John and Biggs, 1994, Rhode and Strumpf, 2003). It can be difficult to work with such models analytically and an *ABM* has appeared (Kollman, Miller and Page, 1997a, b).

The behavior of political parties as they seek to attract voters has been studied at book length by Laver and Sergenti (2011); older work includes Kollman, Miller and Page (1992). An interesting contrast between analytical models and *ABM* is that the latter are characterized by perpetual adaptation and adjustment, not equilibrium.

In sociology a good summary of *ABM* is Squazzoni (2012). Some have called for unifying the discipline with *ABM* (Gintis and Helbing, 2013). An older overview is Macy and Willer (2002). Methodological advocacy for so-called ‘analytical sociology’ (Hedstrom and Swedberg, 1998, Hedstrom, 2005) is very supportive of *ABM*. Collective action has been studied with *ABM* (Macy and Castelfranchi, 1998, Macy and Flache, 2002). Biggs looked at 19th C strikes in Chicago as propagating between factories like a forest fire (Biggs, 2005, Andrews and Biggs, 2006).

Agent-based demography is an active area of research (Billari and Prskawetz, 2003). Local norms of fertility exist and have been modeled with *ABM* (Kohler, 2001).

Agents on *GIS* landscapes are a very active area of research within geography; see Heppenstall, Crooks, See and Batty (2012) for an excellent overview. The complex systems perspective on urban dynamics (Batty, 2005, White, Engelen and Uljee, 2015) uses both *ABM* and *CA* approaches.

Within anthropology there are a number of researchers using *ABM*; see Kuznar (2006) for a somewhat dated overview. These models are typically quite data intensive, often with environmental and archaeological data extending over hundreds or thousands of years. Early examples include (Lansing, 1991) and Axtell et al. (2002) and an early *ABM* research monograph in this area is Kohler and Gumerman (2000). A recent survey is Cegielski and Rogers (2016).

III State of the *ABM* Art

The number of researchers who employ agent computing in their work has grown exponentially over the past decade. Today well in excess of 10,000 papers per year are published annually across the *ABM*, *MAS,* and *IBM* communities with no sign that growth is tapering off.[27] Indeed, as software for creating agent models matures, as researchers become more computer-savvy, and as hardware capabilities expand, it has become easier than ever to build and experiment with agent models. Here we will look here at the contemporary literature on agents in economics and finance and describe the main features characterizing the work, focusing in particular on the ways in which agents may be able to enrich analytical and empirical research in economics and finance.

The *ABMs* summarized in the section II and the ones appearing today break down into roughly three types: (1) purely theoretical models at the approximate level of abstraction as standard economic theory, with little or no empirical ambition, illustrating some particular mechanism or shedding light on some qualitative, stylized fact(s); (2) models that are capable of quantitatively reproducing aggregate economic data, and which serve to link micro and social levels; (3) models closely related to micro-data, in which the behavior of individuals is quantitatively identified (calibrated or estimated) for purposes of recreating important social phenomena. For instance, among the early models reviewed in section II, the Schelling segregation, El Farol bar attendance, and Santa Fe stock market models all fall into the first category, while later generation financial market models, Axtell's model of firms, and *EurACE* all fall into the second. Few of the early models reached the third level, although some clearly tried, e.g., the Darley and Outkin (2007) *NASDAQ* decimalization work and the experimental-computational toils of Duffy (2001). While the first kind of *ABM* vastly outnumbers the other two types, probably because it is easier to build, explore, and publish, what is different about the current crop of models is the growing number that fall into the second and third categories. But there remains much interest in what are largely theoretical, 'proof-of-concept' *ABMs* of the first type, frequently for relaxing the standard idealizations of table 1.

A Heterogeneous Agents

There is a large literature pointing out the limitations of representative agent models generally and in macroeconomics specifically, e.g., Kirman (1992). The notion of using a representative entity in economic models is, ostensibly, due to Marshall (1920) who invoked the 'representative firm' as an analytical expedient. But by now it has acquired the status of an outright methodology, a second-best solution to analytical difficulties associated with solving equations when more than two agents are interacting.

The penalty for this simplification is a loss of information about distributional features that would generally be useful to have. Sometimes simple distributional issues can be added to an analytical model by considering multiple agent types but this may not be satisfactory for many purposes.

Heterogeneity can be both exogenous and endogenous. There is enormous diversity in

[27] The penetration of *ABM* into the economics, finance, and the social sciences today looks a lot like the diffusion of game theoretic ideas into these fields over the past 70 years—initial appearance, followed by a decade or more of low adoption, then an acceleration leading to 'take off' and exponential growth. Experimental and behavioral economics have gone through their own explosive growth in between the rise of game theory and the emergence of *ABM*, as discussed in detail in section V below. Interestingly, bibliometric analyses of these distinct communities suggest that there is very little overlap between them (Niazi and Hussain, 2011) while some have called for more (Wellman, 2015).

human populations, of course, whether of genetic or phenotypic in origin, both physical and cognitive, as expressed behaviorally and measurable parametrically. With *ABMs* the many dimensions of *exogenous* population heterogeneity can be represented to any degree justified by data, and software for creating synthetic populations, grounded in data, has begun to appear (Adiga et al., 2015). Then, with appropriate behavioral repertoire in place for the social or economic process being investigated, further *endogenous* heterogeneity is usually produced.

The systematic incorporation of agent heterogeneity into *ABMs* is a common motivation for building such models today. Often heterogeneous agent *ABMs* hew quite closely to conventional specifications otherwise.[28]

A generation ago the Nobelist Phil Anderson (1972) pointed out that there was a class of problems in physics in which increasing the scale of a system resulted not simply in a larger system, but potentially a system having different behavior. Thus his motto 'more is different,' a kind of endogenous heterogeneity that can develop at the aggregate level. A new trend in *ABMs* is to work with large-scale models. Such models can often have the property that more agents produce different phenomena. For example, in Axtell's model of firm dynamics there is no way to get realistically large or long-lived firms without realizing large-scale models. Without long-lived firms there is no way to get long job tenure, and lots of other properties of real firms. If you want to study the emergence of million worker firms, and produce the kinds of aggregate economic fluctuations induced by such firms (Gabaix, 2011), you need to have a very large number of interacting agents.

B Limited Information, Bounded Rationality

Another very common motivation for *ABMs* is the desire by researchers to move beyond rational agents, whose uncertainty is specified neatly as independent stochastic realizations from well-behaved statistical functions. For many years rationality specifications have been under widespread revision from behavioral economics (1978a, b, Kahneman and Tversky, 1979, Tversky and Kahneman, 1981, Slovic, Fischoff and Lichtenstein, 1982, Tversky and Kahneman, 1986, 1997a, b, c, d, Gigerenzer, Todd and Group, 1999, Gigerenzer, 2000, Gigerenzer and Selten, 2001). However, rationality persists as the default in much of economic theory, presumably largely for reasons of analytical intractability since experimental results on how people really behave typically do not have simple mathematical structure (Simon, 1998). Because *ABMs* do not face this same constraint, it is usually straightforward to incorporate behavioral specifications directly into computational models.

Computer scientists working with *MAS* have demonstrated a strong interest in rationality (Sandholm, 1999, Shoham and Layton-Brown, 2009, Parkes and Wellman, 2015). However, as a practical matter, full-blown rationality is often very difficult to implement in agent models, due to computational intractability (Papadimitriou and Yannakakis, 1994). At the agent level, figuring out how to act rationally is computationally hard (Daskalakis, Goldberg and Papadimitriou, 2006, 2009), while at the aggregate level the computational job of the Walrasian auctioneer or the Hurwiczian mechanism designer is provably among the hardest problems in all of computer science (Hirsch, Papadimitriou and Vavasis, 1989, Papadimitriou, 1994, Conitzer and Sandholm,

[28] The next volume in the *Handbook of Computational Economics* series (volume 4, edited by Hommes and LeBaron) focuses on heterogeneous agents and will be a good source for work in this area.

2002); for a review of the complexity of computing economic equilibria see Roughgarden (2010). A related problem with rationality has to do with the difficulty of predicting opponent behavior (Foster and Young, 2001), although if playing anonymously in a large population the problem is easier (Kearns and Mansour, 2002). Summarizing the last two paragraphs rationality makes analytical models easier to work with than behavioral alternatives, while the reverse seems to be true for *ABM*.

However, there is a further motivation for *ABMs* at work here. Real economies are decentralized in deep and important ways (Hayek, 1945, 1964), making information not just diffuse but also tacit (Polanyi, 1948). This means that simply giving agents the ability to act in probabilistically uncertain economic environments is a very narrow formulation of uncertainty. For in reality there will be wide swaths of knowledge having to do with production, distribution, pricing, and so on, that are not available to all or even most agents. This kind of dispersed information is hard to represent in a satisfactory way analytically. However, in *ABMs* agents essentially always glean information from their local environment, perhaps supplemented by some gross aggregate data, and make decisions on the basis of this dynamic, idiosyncratic set of 'facts' which constitutes their knowledge. Under what conditions does it make sense to acquire more knowledge, or the jettison old information in pursuit of better performance or outcomes? These questions have not been systematically dealt with in economic theory but are often explicitly present in *ABMs*, although have not, by any means, been dealt with successfully.

Sims (1980) argued that beyond rationality there is a 'wilderness of bounded rationality'. What we have today in the *ABM* literature is a spectrum of approaches to agent sophistication, all of which qualify as one form of bounded rationality (*BR*) or another. Unsurprisingly, different kinds of *BR* seem useful in distinct contexts. It is as if we can now see that the forest is made up of individual trees, but these trees are not each completely unique, rather there are a variety of species. The main types of agents commonly employed in *ABMs* are briefly described next, from simple to sophisticated.[29]

1 Simple (myopic/reactive/adaptive) agents

More than 50 years ago Becker (1962) demonstrated that randomly behaving buyers could create downward sloping demand curves. While completely random behavior might be an interesting lower bound, more recent work in this tradition has focused on simple agents who might use some form of randomization in their behavior but who are *purposive*, i.e., they have some facility for judging the welfare consequences of their actions. Such *adaptive* agents typically do not have detailed internal models of their environment. Rather, they tend to be *myopic* and are commonly referred to as *reactive* agents in the *MAS* computer science literature (Weiss, 1999). In economics and finance such agents are often called 'zero-intelligence' (*ZI*) agents (Gode and Sunder, 1993, 1997), although this name can be a source of confusion since it suggests behavior that is completely random, which is not typically what *ZI* agents do. For instance, in simple market environments *ZI* sellers will try to find buyers in order to cover the cost of the goods they are selling, while *ZI* buyers will not pay more for a good than they believe it is worth. But the determination of exchange prices between the buyer bid and the seller

[29] Chen has written a history of agent types in use in *ABM*, thus permitting brevity here (Chen, 2012). The standard textbook in *AI* (Russell and Norvig, 2010) also takes an agent-centered approach and includes several kinds of agents that have, historically, had little application in economics, e.g., logical agents.

ask is often modeled as being random in some sense. In many cases simple adaptive agents can produce high performance, particularly in the vicinity of an equilibrium (Lucas, 1986), making their study relevant to more traditional rational solutions, but it is also known that there are a variety of contexts in which very simple may not perform well. For example *ZI* agents do not do well when supply and demand curves have unusual shapes, although rather slight modifications to their behavioral specification can produce much better performance, e.g. zero-intelligence plus (*ZIP*) traders (Cliff and Bruten, 1997a, b). *ZI*-type agents are closely related to 'probe and adjust' agents (Kimbrough and Murphy, 2009, Huttegger, Skyrms and Zollman, 2012).

A closely related class of *simple*—myopic yet purposive—agents appear in game theory. These so-called 'low rationality' agents attempt to figure out how to behave by doing simple things like 'best reply' to their strategic environment (Young, 1993a, 1998). There also exist 'best reply to best reply' strategies, 'best reply to best reply to best reply' and so on. Example *ABMs* include Axtell, Epstein and Young (2001). Such strategies are often accompanied by noise so that players select random strategies occasionally as well. Best reply to best reply suggests discrete cognitive 'levels'. *K*-level cognition (Camerer, Ho and Chong, 2004) has found use in *ABMs* (Latek, Kaminski and Axtell, 2009).

In summary, simple purposive agents—perhaps the simplest ones of significant interest—behave by, in essence, taking their environment as fixed and reacting in their own immediate self-interest, by adapting their behavior to their immediate circumstances. If they find that additional adaptations might improve their welfare in subsequent periods they do this as well. Basically, they follow utility or profit or payoff gradients. Over time they can effectively learn but they do so without a formal model of their environment.

2 Agents who learn (formally)

When agents have models of their environment they are capable of formal learning. There are at least three broad classes of learning discussed in the voluminous literature on this topic. *Individual* learning is typically treated as a single agent gleaning data from its environment and updating its model of the environment. It has roots in decision theory, as a game against Nature. *Social* learning concerns multi-agent situations in which individuals build models either of the population of other agents or of individual agents. This kind of learning can involve strategic dimensions while such considerations are normally absent from individual learning. Finally, *group* learning refers to how and what individuals learn in order to behave as a group for the good of the group. This is more common in biology than the social sciences—think fish schools (Couzin et al., 2011, Miller, Garnier, Hartnett and Couzin, 2013, Kao et al., 2014) or flocks of starlings (Hemelrijk and Hildenbrandt, 2011)—but occasionally appears in *MAS* computer science.

There is much work today on individual learning in behavioral and experimental economics as well as in the *ABM* and *MAS* research communities. In each area there is a wide variety of learning formalisms in use. Reinforcement learning, cue learning, probably approximately correct (*PAC*) learning and other schemes common to *MAS* have been reviewed (Shoham, Powers and Grenager, 2004, Panait and Luke, 2005). In experimental economics Erev-Roth learning (1995) and experience-weighted attraction (*EWA*) learning (Camerer, Ho and Chong, 2002) are well-known and have been surveyed (Camerer, 2003). Excellent reviews of *ABM* learning by Brenner (1999, 2006) make it unnecessary for us to rehash this literature further here.

Some early work in *ABM* learning focused on evolutionary algorithms (Arifovic and Eaton, 1995, Bullard and Duffy, 1999, Dawid, 1999), although lacking a strong basis as individual learning such methods are often interpreted as population-level (social) learning, or perhaps simply as a way to incorporate optimization into *ABM*. Learning at multiple levels may amplify the complexity of economic and financial phenomena.

For a while it was common to use neural networks inside agents (LeBaron, 2001a), although this appears to be more rare today. So-called 'deep learning' is the new kid on the block in the learning world (LeCun, Bengio and Hinton, 2015, Schmidhuber, 2015). Exploitation of such models has begun and it is too early to know the implications.

It is interesting to compare approaches taken by economists to those of computer scientists when it comes to learning. About a decade ago in a special issue of the journal *Artificial Intelligence* (Vohra and Wellman, 2007), researchers from both disciplines wrote about the topic. The economists were mainly interested in learning schemes that led to Nash equilibria (Erev and Roth, 2007, Fudenberg and Levine, 2007, Young, 2007) while the computer scientists (Sandholm, 2007, Shoham, Powers and Grenager, 2007, Stone, 2007) were asked where computationally plausible learning rules led.

3 Behavioral agents

ABMs in which agent behavior is made to reproduce the results of experiments are growing in number. There are *ABM*s in which agents behave in accord with prospect theory (Kahneman and Tversky, 1979, 1992) including the previously mentioned EurACE model, others where agents engage in hyperbolic discounting, and yet others in which agents possess one or more behavioral biases in their decision calculus.

There is a relatively long history of building the behavioral specifications of *ABMs* from experimental data, as reviewed by (Duffy, 2006), assertions to the contrary notwithstanding (Wunder, Suri and Watts, 2013). For instance, Cotla (2016) has built *ABMs* to reproduce laboratory experiments, then perturbed the experimental set-up and the *ABM* in non-trivial ways, running the computational model in advance of the actual experiment to forecast the likely outcome, then comparing the result with human subjects directly to the computational results and finding good agreement.

Many of the behavioral specifications that have come out of laboratory experiments are not readily tractable analytically so computation may be the natural way to proceed (Simon, 1998). *ABM* is also good for 'scaling-up' laboratory results to realistic population sizes, to look at side effects, unintended consequences, etc. Surely there will be more use of behavioral and experimental results in *ABMs* going forward.

4 Cognitive architectures for agents

Originating roughly contemporaneously with the behavioral revolution in economics was the field of cognitive science within psychology (Newell and Simon, 1972). Indeed, some of the founders of cognitive science were involved with economics.

Beginning in the 1980s computational implementations of cognitive theories began appearing as so-called cognitive architectures (Anderson, 1983), an early version of which was *SOAR* (Rosenbloom, Laird, Newell and Orciuch, 1985, Laird, Rosenbloom and Newell, 1986). *SOAR* has been used in a variety of high-fidelity simulation environments, such as military and civilian air traffic, typically involving one or just a few humans in software. *ACT-R* is a more recent computational architecture (Anderson, Matessa and Lebiere, 1997) and agent systems have been built using it. More recently, several other computational models of cognition have appeared, including some focused on social behavior (Dautenhahn, 1999) and more suitable for *MAS* and *ABMs*. The book by Sun (2006) is a good overview and describes his *CLARION* architecture.

Although technically not a cognitive architecture the so-called Beliefs-Desire-Intention (*BDI*) architecture is a non-human model of cognition that has been very popular in the *MAS* community (Rao and Georgeff, 1995), less so in *ABM*. Its structure and performance have been contrasted with that of SOAR (Sun, 2006).

None of these models have deep biological grounding. A relatively new kind of cognitive model has components that resemble biologically structures. These are called biologically-inspired cognitive architectures (*BICA*), and a number of them have appeared; see Goertzel et al. (2010) for a review. So far they have been little used in *ABM*.

Many of these 'rich' cognitive models face challenges of intelligibility. For when some social phenomenon arises in models featuring many agents, all with deep cognition, it is not clear whether it should be thought of as a consequence of the cognitive model, of the social interactions, or both. Of course, this same difficulty haunts all of the empirical social sciences, so in confronting it *ABMs* are simply recapitulating the real world.

One interesting conceptual issue that arises with the current generation of cognitive models involves foresight. Building agents that make predictions about their social world is a difficult task and presumably people use heuristics to deal with this problem. (Indeed, the whole reason we are building *ABMs* in the first place is because *social* science is hard!) Gross simplifications, such as giving all the agents the same expectations, or specifying that they all have certain propensities, is one solution. So it is difficult to outfit artificial agents with foresight for precisely the same reason it is hard to make predictions in the real world: because economies are *CAS*! Precisely because it is hard we should expect agents (and the people they represent) to take varied approaches to the problem, adding further complexity through heterogeneity (Izumi, 2001, 2002). The final point to make in this regard is that a 'veil of complexity' thus hangs over all attempts by individual agents for forecast the future, which would seem to radically attenuate the relevance of the 'Lucas critique,' at least for models having non-trivial levels of social complexity. To put this another way, given that one reason we turn toward *ABM* is because of the complexity of the economy, it may make sense for some of the agents in our models to have *ABM*s running around in their heads. In principle such computations are possible (see section H below), although rarely executed in practice.

5 Emotional agents

The role of emotions in decision making has been long noted (Hume, 1896 [1739]) and has been studied with regard to economic decisions in particular (Frank, 1988). However, it is fair to say that such models are not part of the mainstream of economics today, nor have they received the attention they are due from the *ABM* community, with a few exceptions (Epstein, 2013). However, a variety of models for the role of emotions have appeared within the *MAS* literature (Elliott, 1992, Bates, 1994, Velásquez, 1997). Sometimes these are grounded in data while other times they are more notional. In any case, more work needs to be done on this topic.

6 Beyond utilitarian agents

It is almost universally true that models in economics and finance make use of utilitarian agents, i.e., who pursue their own self-interests. Indeed, this is how we define *purposive*—agents who seek higher rewards or performance for themselves, sometimes (perhaps even often) to the detriment of their peers.

However, there are other ways to motivate agents and a small but growing body of research, especially in *MAS*, investigates *deontic* agents, i.e. individuals who follow prescribed rules and norms, recognize duties and obligations, are willing to 'see to it that…," and will generally subordinate their self-interest to group goals. Now it turns out that there are important relations between utilitarian and deontic agents under certain conditions (Horty, 2001). Research on deontically-motivated agents has uncovered a set of modal logics known as *KD45* that have many 'nice' properties vis-à-vis human behavior (Lomuscio and Sergot, 2002). A closely related area is doxastic logic, or reasoning about beliefs. An early survey of these topics is (Wooldridge and Jennings, 1995a). These are active research areas. Danielson (1992, 1996) considers moral agents.

C Direct Agent Interactions

In many economic models agents do not interact directly with one another but rather decide how to behave using aggregate economic quantities like prices, interest rates, and wage levels. That is, whether in representative agent models or truly heterogeneous agent models, such as one encounters in general equilibrium, it is not conventional for agents to glean information from their peers, to communicate in any meaningful way with anyone, or consummate economic exchange directly with each other. It is a kind of methodological individualism without individuals! This abstraction does little violence in static equilibrium settings, since the *mechanisms* by which fixed points are assumed to be achieved are not studied. That is, *substantive* rationality abstracts from the details of who trades with whom, or where information comes from. However, when one attempts to 'grow' economic phenomena using *ABM*, the notion of direct agent-agent interactions comes quite naturally. In this section we focus on the varieties of such interactions, first with respect to their topology and then manner in which agents are activated to interact.

1 Networks

In the last 15 years there has been a great flowering of the science of networks (Watts, 1999, Barabasi, 2002, Newman, 2010) with social (Jackson, 2008) and physical networks (e.g., Atalay, Hortascu, Roberts and Syverson, 2011) earning their distinct places within economics. Networks are not part of the standard neoclassical picture, as suggested in table 1 above, as the default assumption in economics is that agents are 'well-mixed'—each sees the same prices, interest rates, wages, and so on. Relaxing the completely connected character of neoclassical agents with a network of interactions was attempted early on by Föllmer (1974) and discussed in detail by Kirman (1997). *ACE* models usually take networks into account in one way or another. A recent monograph reviewing the literature on agents and networks (Namatame and Chen, 2016) relieves us from having to review this large literature here. An important motivation for agent computing and social networks is that realistically complex networks are often difficult to work with analytically, making recourse to *ABM* natural (Jackson, 2008: 406-7).

2 Agent interaction regimes

A facet of direct agent-agent interactions beyond networks involves agent activation. It is conventional practice in *ACE* models to permit only one—or at most a few—agents to be active at any one time. In part this stipulation derives from the serial nature of the computer hardware on which agent models are typically executed. But it also stems from the desire to not engage in perfectly synchronous updating, as is common in cellular automata. Perfect synchrony should be avoided because (1) it can lead to the production of meaningless artifacts in model output, as we saw in the Nowak and May versus Glance and Huberman affair, and (2) the social world is clearly asynchronous.

One way to model complete asynchrony, and a high degree of agent autonomy, is to put each agent on its own thread of execution. This is rarely done today, due to the difficulties of writing highly parallel code. Rather, nearly all *ABMs* (>99%) are single-threaded, making agents only partially autonomous. With all agents running on a single thread which should move first, which second, and so on. The order of execution can be randomized in various ways, and one might hope that the overall results of a model would not depend on such microscopic details, but that is generally *not* the case.[30]

While a minority of *ABMs* uses some endogenous internal model state to activate agents, such as when a profit opportunity is sensed, most do not. Rather, most models, in effect, generate a schedule of agent activations, usually stochastically. There are three ways this is commonly done (Axtell, 2001). First, in a population of A agents, *uniform activation* is the process by which all A are activated once, sequentially, in effect defining a unit of model time or a period. Over p periods each agent is activated p times. In order to insure that no artifacts are produced from the i^{th} agent always moves before the $(i+1)^{st}$, the order has to be partially randomized regularly. Usually this can be done efficiently.

Contrast this with a second activation regime, called *random activation*, in which a period is still defined as A agents being activated, but at each instant within the period the agent selected to be active is random. This means that during any particular period some agents may get turned ON more than once while others do not get activated at all. In essence, the activity of any particular agent in a period now has some variability where it did not for uniform activation. This subtle difference is known to matter in some *ABMs* (Axtell, Axelrod, Epstein and Cohen, 1996, Lawson and Park, 2000).

The third most common agent activation regime is *Poisson clock activation*. In this scheme each agent is given a series of times—a schedule—when it will be active by drawing from an exponential distribution, meaning that the times between activations are independent. The activation times of the whole population are then sorted into a *master schedule*, used to determine which agent moves next. This type of activation produces A activations/period *on average*, and within each period some agents can be more active than others, as in random activation. This regime has been used by game theorists, since it has tractable analytics (e.g., Lagunoff and Matsui, 1997).

Other activation schemes are possible (Comer, 2014) but the three above are the most common ones used in *ABM* today. They are compared in table 4. Today we have precious little understanding of how to select between these distinct activation regimes. In the best of all possible worlds we would have empirical data on the character of agent activity, but such data are virtually non-existent today.

Activation Regime	***# active/period***	***activations/agent/period***
Uniform	A	1
Random	A	1 on average
Poisson clock	$\bar{A}$	1 on average

Table 4*: Comparison of common activation regimes for a population of* A *agents*

[30] Although it can be proved to be the case in some special circumstances (Chen and Micali, 2013).

The activation schemes above are imposed on the agents exogenously. It is also possible to have *endogenous* activation in which quasi-autonomous agents decide themselves when to be active and then put in a request (to a central authority or to the operating system) to be activated. Such activation schemes are used in certain market models in which agents decide when to enter bids.

All of the above are highly democratic activation schemes. Considering agent activation to be a scarce resource, in the sense that there are only so many *CPU* clock cycles to be allocated to all agent activations, some (e.g., Page, 1997) have wondered what might happen if agents cold 'buy' or otherwise acquire additional activation cycles. By analogy, when a firm hires a worker in the real world it is essentially purchase 8 hours/day of effort that it would not otherwise have at its disposal. While it is fair to say this idea for valuing agent activation has not found its way into many models to date, it does seem like a fertile idea, deserving closer study.

D *ABM* Markets: Beyond the Walrasian Model

Markets are workhorses in *ACE* models, unsurprisingly. These come in all shapes and sizes, from *CDAs* to bilateral trading, with 2 to 2 million goods, either divisible or indivisible, and so on. But what is perhaps surprising is that it is rare for such market specifications to hew very closely to the conventional Walrasian picture of markets. One reason for this is the computational intractability of such market mechanisms, i.e., the computational complexity of Brouwer fixed points (technically in complexity class *PPA*), as mentioned above. While there exist algorithms to approximate these (see fn. 18), they have little to do with markets operating from the bottom up (Rust, 1997).

Given this state of affairs, the kinds of markets that appear in *ABM* often feature 'messy' things like local prices, miscoordination, and inefficiency, at least initially (Moss, 2001b). But as *ABMs* 'spin forward' in time they can often reduce these problematical features: prices become more homogeneous as *MRSs* get aligned, agents progressively coordinate their behavior, and inefficiencies decline. But it is rare to find anything like the perfect, noiseless, crystalline world of Walrasian prices and allocations.

A consequence of having local prices in many *ABMs* is that large volumes of trades take place at prices that are different from the Walrasian ones that could, in principle, be computed by a social planner having perfect information. Such trades have welfare effects, meaning that the utility levels produced in such decentralized, distributed markets are generally *not* Pareto optimal. At first blush this departure from the welfare theorems may appear to limit the value *ABMs* for theoretical or other work. However, the spontaneous appearance of price heterogeneity can be thought of as a *feature*, not a *bug*, for price dispersion is common in real-world markets and economies, at least those outside of authoritarian regimes where uniform prices are handed down from on high. Prices can fluctuate over time and vary over space, both in the real world and in *ABM* models of it. This is OK, as long as we can make the price dispersion in the model reflect that found in the real world, as price heterogeneity breaks the complexity barrier.

E Institutions, Emergent

A common complaint against general equilibrium theorizing is that it is 'institution-free' (e.g., Leijonhufvud, 1993). The essence of this critique is that real-world economies feature a variety of institutions for either formally or informally coordinating activity. In a fully satisfactory economic model a wide spectrum of such institutions would arise, or at least be present, acting to coordinate economic activity, at least partially.

Today we have very incomplete knowledge of which rules of agent behavior are sufficient to produce realistic-looking multi-agent institutions. However, there are several agent models in which certain intermediate level, multi-agent conventions and norms arise (e.g., Kandori, Mailith and Robb (1993), Young (1993a, b), Bicchieri and co-authors (1993, 1997), Shoham and Tennenholtz (1997). For example, in models of agricultural share-cropping (Young and Burke, 2001) social conventions associated with contracting are the usual way harvests get divided. While there is today no satisfactory theory of such emergent institutions, some progress has been made (Young, 1998).

An example of this in a pure *ABM* setting is Axtell's (1999, 2002, 2016) firm formation model, While multi-agent firms are permitted to form in this model, the overall size and structure of the population of firms can be thought of as emerging from the interactions of the agents. Under specific conditions there can arise very large firms, having millions of workers, say, and once these are produced they very much alter the landscape of employment opportunities that are available to other agents in the population. That is, once this structure has emerged from the bottom up, it then has important ramifications from the top down for subsequent epochs of the economy.

In order for an institution to be considered emergent it is necessary to describe a mechanism that produces it. Some have claimed that the quintessential example of emergence in economics is Adam Smith's invisible hand, and the corresponding welfare theorems of general equilibrium (Durlauf, 2012). But today we do not know how to 'grow' Walrasian prices from the bottom up, so there is no basis on which to call it emergent.[31] Indeed, due to the computational problems previous mentioned, there is strong reason to doubt that it can be emergent, unless additional structure is added.

F Economies as Many-Level Systems

It is conventional for economists to consider economies as multi-level systems, to treat the agent level as different from the aggregate level. In moving between levels one must be careful not to succumb to the dual fallacies of composition and division. It is a truism in economics that knowing how the micro (agent) level works may not give us much insight about the operation of the macro-level, an explicit acknowledgement of the *fallacy of composition*. But explaining aggregates in terms of individuals is all too common, as in the anthropomorphism of stock market fluctuations. Likewise, attempting to draw conclusions about the agent level from aggregate data is equally problematical, a *fallacy of division*, closely related to the ecological inference problem in econometrics. Agent computing offers a way to explore the multi-level character of economies by permitting higher order structures to emerge from lower level interactions.[32]

For example, consider again the emergence of firms from the decisions of individual workers (Axtell, 2016). In general equilibrium theory it is normal to consider firms as occupying the same ontological level as consumers. But it is useful to consider them as 'above' the agent level, since they are composed of multiple agents. They occupy a meso

[31] For Nozick (1994),'invisible hand explanations' required credible underlying social processes.

[32] Interestingly, within the *MAS* research community, populated as it is mostly by computer scientists and engineers, there is strong difference of opinion as to the value of *emergence* in agent models. This was clearly on display in the inaugural edition of a new journal in which some of the editors thought that focus on emergent behavior was warranted and part thought it was not (Jennings, Sycara and Wooldridge, 1998).

level, with the aggregate level, involving the entire population of firms, above. This three level picture of an economy is shown in figure 5. Considering the aggregate state to be z while the meso-level is y. An *ABM* operates solely at the lowest level x, and its code represents the function f for marching the model forward in time. As to whether and how one might derive the functions g and h for faithfully representing the higher levels, this is an important question for industrial organization, say, and macroeconomics, respectively. But note that these functions are implicit in the *ABM* in the sense that at each instant in time we can simultaneously 'see' x, y, and z, and if we wait a bit we can inspect them again in the next period. Real economies may be 4, 5 or 6 level systems when the many levels of regulation and governance are taken into account.

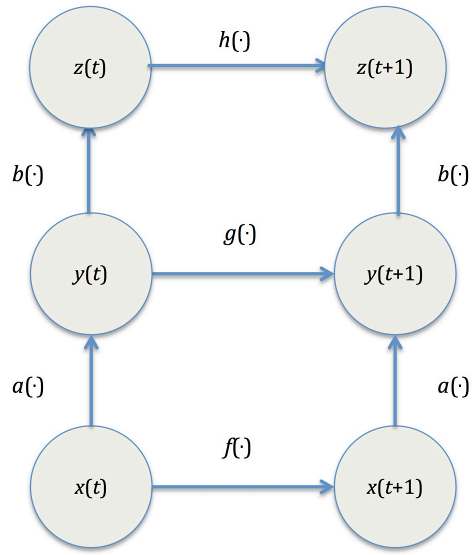

Figure 5*: A three level economy with* x(t) *the micro-level,* y(t) *the meso-level, and* z(t) *the macro level, with aggregation functions* a *and* b *mapping states from micro to meso and meso to macro, respectively*

G Social Steady-States With or Without Agent-Level Equilibrium

When agents are adaptive and their environment is changing, or when agents are learning how to alter their behavior to take advantage of their circumstances, there will generally be changing behavior at the agent level. Perhaps only a few agents are modifying what they are doing, or a majority of the population. In any case, changes in behavior may indicate that the agent level is out of equilibrium. This situation is very common in *ABM* (Arthur, 2006). In fact, in wide classes of economic and finance *ABMs* it is normal to observe sizable fractions of agents changing their behavior regularly. Perpetual adaptation and adjustment is the norm in *ABM* and this may or may not lead to systemic changes at the aggregate level. Such changes may resemble mixed strategy Nash equilibria, as we saw in the El Farol model, but they need not.

Equilibrium at the agent level, e.g., Nash or Walras, certainly implies equilibrium at the aggregate level, i.e., it is *sufficient* (Farmer and Geanakoplos, 2009). However, it is not *necessary*, for if there are dynamics among agents that 'cancel' each other out then we could expect aggregate stationarity to arise, social steady-states. In practice, most *ABM*s exhibit flux at the agent level yet stable patterns and statistics at the aggregate level. Mathematically it would be useful to have 'solution' concepts that permit agent dynamics and population patterns of this type. These remain to be developed.

One way to think about all this, particularly given the content of the last two sections, is that macro-steady-states are emergent phenomena. What can we say about them ahead of time? Can we deduce things about their size or stability? We are reductionists and these emergent configurations are produced by the actions of the agents, of course. But we are also pragmatic anti-reductionists (Simon, 1996 [1969], Faith, 1998), and just as it is hard to determine what will emerge in the 'Game of Life' simply by looking at the rules, it is very hard to forecast the character of emergent steady-states in the economy.

H Implementation of *ABMs*

By far the most common question concerning *ABM* we get is 'What software should I use to build my model?' This question has many facets and picking the wrong software for a project can be disastrous. Here we provide some guidelines based on current technology. Happily, there are good comparisons of existing software packages—Kravari and Bassiliades (2015), supplementing older ones of Gilbert and Bankes (2002) and Dibble (2006)—meaning we can be brief, editorializing a bit based on our experience.

There are essentially four distinct ways to create an *ABM*, (1) code in a native programming language like *Java* or *Python*, (2) write your model in a mathematical or statistical environment like *MatLab*, *R*, or even *Mathematica*, (3) code your model using a software framework for *ABM* like MASON or *RePast*, or (4) create your model in a high-level, *ABM*-specific software environment like *NetLogo*. Each of these systems has advantages and disadvantages, so selection involves trade-offs. Specifically, the *lower* the number on our list the faster your model will probably run, eventually, once it is successfully coded and debugged. However, the coding and debugging time declines as the number on our list gets *higher*. For example, native *Java* code is going to run much faster than *NetLogo* code but it might take you significantly longer (2-10x) to get a non-trivial model up and running in *Java* than in *NetLogo*.

Empirically, most *ABM*s used for research in economics are built in *NetLogo*, *MASON*, or *RePast*. These are each mature systems with large user bases, reasonable documentation, and performance good enough to use for research. In finance it is probably the case that more than half of all *ABMs* are created in *MatLab*. This is because that system is designed to high-performance numerical computation and is especially suitable for solving equations—agents in finance *ABM*s often have to solve portfolio optimization, arbitrage, and other mathematical problems in determining how to behave. We summarize the characteristics and performance of these four systems in table 5 where we also include *Mathematica*, not because it is widely used for *ABM* but because many economists use it. Software systems rarely used for research *ABM* include SWARM (Minar, Burkhart, Langton and Askenazi, 1996, Terna, 1998, Luna and Stefansson, 2000, Stefansson, 2000), CORMAS (LePage et al., 2000), and AgentSheets (Repenning, Ioannidou and Zola, 2000), and we will not say more about these here.

Software	***OOP?***	***Programming***	***Compiled?***	***Animations?***	***Speed***	***Max agents***
NetLogo	yes	own language	no	yes	poor	10,000?
MASON	yes	*Java*	byte code	yes	good	1,000,000
RePast	yes	*Java*, *C#*	byte code	yes	good	1,000,000
MatLab	partial	own language	can be	yes	good	100,000
Mathematica	partial	own language	can be	slow	poor	1,000?

Table 5: *Comparison of software environments for creating* ABM

For each, a few points of clarification and one or more references to the literature. *NetLogo* (Wilensky and Rand, 2015) combines a programming language (having hybrid *OOP* and functional features) with an highly configurable development and analysis environment. It is excellent for rapid-prototyping but too slow for large models. *MASON* (Luke et al., 2005) is based on Java and requires users to code in that language. It has excellent analysis and visualization interfaces. *RePast* (North, Collier and Vos, 2006) users code their model in either *Java* or *C#*. It has many features in common with *MASON*. *MatLab* has object extensions but they were added somewhat recently and are often not used by people writing *ABM* code. Its performance is good. Objects are not a natural part of *Mathematica* but the functional aspects of its programing language means that *ABM*s can be written *very* compactly. For instance, Gaylord and D'Andria (1998) have programmed the Schelling model in 5 lines of *Mathematica* code! However, it tends to be slower than the others in execution of *ABM*.

A new approach *ABM* deserving brief mention is by programming the video boards that are part of all modern microcomputers. These so-called graphics processing units (*GPUs*) have greatly improved their performance with progress in video game technology. D'Souza, Lysenko and Rahmani (2007) programmed the *Sugarscape* model to run 1,000,000 agents at 25 frames/second while only a few hundred agents could run at that speed when the model was first created (Epstein and Axtell, 1996). *FLAME* is an *ABM* programming environment designed specifically for *GPUs* (Kiran et al., 2010).

I Empirical Grounding of Agent Economies

There are a wide variety of ways that *ABM*s attempt to represent the real world, and several distinct approaches to making such models reflect reality.[33] Given the multi-level character of *ABM* a model may be empirically-relevant at one or more levels. For example, Friedman (1953) famously argued that a model could be useful at the aggregate level while being behaviorally wrong at the agent level, what Simon dubbed "the principle of unreality" (Simon, 1963). In this section we will briefly review distinct approaches to building progressively realistic *ABM*, roughly following the typology laid out in Axtell and Epstein (1994).

To begin, *ABMs* always need behavioral specifications. Where do these come from? Possibly the model builder has sufficient knowledge of the domain to create reasonable behavioral rules in software, at least up to some unknown parameters (to be estimated). Possibly domain experts can be queried for rules, parameters, or both, a process known in decision theory as *expert elicitation* (Morgan, Henrion and Small, 1990), although care must be taken to avoid certain pitfalls (Morgan, 2014). There are also techniques to infer rules (Thagard, 1988). An *ABM* up and running based on such inputs is thus *plausible* at the agent level. For example, in creating an artificial stock market, agent traders need to be trading risky assets for riskless ones in order for such a model to pass the basic 'sniff'

[33] The phrase 'verification and validation' is common in operations research for questions concerning the veracity of computational models. 'Verification' usually refers to whether a model is logically sound, e.g., free of bugs, and executing in accord with an independent specification of the model—basically, that it is doing what it is supposed to be doing. This is typically a low bar and checked heuristically. Although there are formal methods from computer science that can be brought to bear on such questions, they are usually impractical for realistic *ABM*. 'Validation' is a relatively uncommon term in the social sciences, for in asking whether a model is a valid depiction of a social process, the answer is rarely 'yes' or 'no'. We shall not discuss issues of empirically-grounding *ABM* as 'verification and validation,' believing the phrase to be inferior to simply 'model identification', especially in economics and finance.

test that it is the model it purports to be. Or stated negatively, if a model cannot pass the basic plausibility test it is *not* a good model, Friedman notwithstanding.

1 Agent models *qualitatively* reproduce aggregate patterns

The next level for an *ABM* to pass muster is whether it is capable of producing outputs that qualitatively resemble aggregate data, so-called stylized facts. Typically there are gross patterns in aggregate data that an *ABM* should 'hit' in order to be considered successful. The way this is done with *IBMs* has been nicely surveyed (Grimm et al., 2005) and many of the same considerations apply in the social sciences. Returning to artificial stock markets, models that cannot reproduce qualitative aggregate phenomena like clustered volatility, heavy-tailed return distributions, and log prices time series with little autocorrelation are unsatisfactory empirically, even at a qualitative level.

In practice there seem to be a wide range of approaches for specifying parameters in models of this type. Sometimes they can be approximated from experience, sometimes set from logical, dimensional, or model-specific regularity considerations (e.g., suppliers will not lower their posted prices below their costs), and sometimes they are simply 'pulled out of thin air' as *sufficient* to produce the kinds of aggregate patterns desired.

2 Agent models *quantitatively* reproduce aggregate data

When aggregate patterns are quantifiable then more formal calibration and estimation techniques can be employed. Perhaps the most common approach used in *ABM* for specifying parameters is search of a model's parameter space in order to minimize the difference between model output and the aggregate data, according to some norm. When the parameter space is not prohibitively large then conventional estimation techniques can be used (Heard, 2014, Heard, Dent, Schifeling and Banks, 2015). Indeed, computational techniques created for analytically intractable analytical models, such as 'estimation by simulation' (McFadden and Ruud, 1994) can often be adapted to *ABM*. Such formal estimation procedures are commonly used in financial market *ABM* (e.g., Alfarano, Lux and Wagner, 2005, 2006, 2007). Spatial *ABMs* can also be estimated using such approaches (Hooten and Wikle, 2010). When the parameter space is large it becomes necessary to search heuristically (Michalewicz and Fogel, 2000, Luke, 2013), e.g., via evolutionary algorithms (e.g., Terano, 2007).

3 Agent models *quantitatively* reproduce micro-data

Many of the same techniques can be used when the kind of data that are available are at the individual level. Considerations related to microeconometrics are now in play (Cameron and Trivedi, 2005), such as the Manski critique (Manski, 1993, 1995, 1997). In essence, if data are not gathered to preserve independence and other properties it will not be possible to distinguish selection effects.

One way around some of these problems is to acquire individual-level data from experiments. In sections III.B.2 and III.B.3 we have discussed the use of experimental data to specify agent models (Duffy, 2006, Wunder, Suri and Watts, 2013, Cotla, 2016). This approach has been utilized in finance settings as well (Hommes, 2011).

4 Replication and 'docking' of agent models

Results from *ABM*s are not as transparent as, say, analytical models, so there has been an active current of research in the field aimed at replication. An early example was a 'docking' experiment of the Sugarscape model made to reproduce the results of Axelrod's culture model (Axtell, Axelrod, Epstein and Cohen, 1996). Another example is the reimplementation of the Sugarscape model (Bigbee, Cioffi-Revilla and Luke, 2005). Such replication exercises may turn up bugs in the original code, imprecise pseudo-code descriptions making *quantitative* replication difficult, or non-robustness of findings, when a small change in behavioral specifications, for instance, can lead to dramatically different results (Edmonds and Hales, 2003, Hales, Rouchier and Edmonds, 2003). For example, adding noise (Klemm, Eguíluz, Toral and San Miguel, 2003) to Axelrod's culture model (Axelrod, 1997b, Castellano, Marsili and Vespignani, 2000) breaks it!

5 Some other aspects of empirical *ABM*

For models capable of generating the same kind of data as can be derived from actual economic and financial processes, direct comparisons can be made. That is, regression models that have unknown specification error vis-à-vis the real world but direct, identifiable (at least in principle) specification error in relation to an *ABM*, can be estimated on both the real world data and the *ABM* data and the results compared. Then, as changes to the *ABM* are made to bring the regression parameters into better agreement with the actual data, the *ABM* can is improved. It may also be the case that the data from the real world was collected according to some sampling or incomplete census effort. In this case data gathering from the *ABM* can proceed in a comparable fashion to how it was acquired in the real world. With apologies to the physicist Feynman, the same phenomena have the same regressions.

J *ABMs* for Policy

In section II.E.2 the use of *ABM* by the *NASDAQ* management to assess the effects of decimalization on the operation of their market in advance of its implementation as policy was described. In the future it would seem reasonable for policy-makers to avail themselves of *ABM* technology in order to test in advance which kinds of regulations make the most sense, whether for producing greater social welfare or simply to avoid noxious side effects of untested policies (Helbing, 2012). While the penetration of *ABM* for such purposes into governance institutions in economics and finance has been modest to-date, in other fields policy decisions based on *ABM* have become standard (e.g., in epidemic control, Gemann, Kadau, Longini Jr. and Macken, 2006, Gomes et al., 2014). Some central bankers have called for new kinds of models to help them manage better.[34]

IV Future Opportunities and Challenges for *ABM*

In this section we discuss areas where *ABM* is poised to make progress and others where there appear to be significant roadblocks, where the limitations of extant *ABM* methodology are limiting further advances, and basic research appears to be needed.

[34] Thus Trichet (2010): *When the crisis came, the serious limitations of existing economic and financial models immediately became apparent….Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools….We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. We need to entertain alternative motivations for economic choices….Agent-based modeling dispenses with the optimization assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention.*

A Opportunity: Micro-Data Integration (Including Social Network Data)

We discussed multiple *ABMs* in which micro-data were essential to the operation of the model. When such data are available, whether from administrative records, customer information, *GIS* layers, or elsewhere, *ABM* can serve as a platform for integrating them.

Social network data represent a case in point. Twenty-five years ago neither *ABMs* nor social network analysis (*SNA*) were in the vocabulary of economists. A lot has changed since the early 1990s. It is our experience that many people working at the research frontier tend to think either in *ABM* or *SNA* terms but few do both. Thus, independent software ecosystems have grown up around these two distinct approaches and overall, there is very little integration, today. Surely this will change. A decade ago there was very little integration between *GIS* software and *ABMs*, but now most of the major agent systems are able to read shape files, making true spatial modeling possible.

B Opportunity: Moving To Large-Scale and Full-Scale Models

An unusual property of *ABM* is its small compile-time, potentially large run-time character. That is, it is normally a relatively simple matter to expand an agent model to fill available memory by simply instantiating more agents. In many circumstances there may be only vanishing value to adding another agent to an *ABM*. However, it may also be that certain kinds of models can profit considerably from larger numbers, for both qualitative and quantitative reasons. Larger numbers of agents can generate qualitatively different behavior—Anderson's (1972) 'more is different' idea—and this may be important in establishing the empirical relevance of an agent model. Larger populations also produce more robust statistics.

Indeed, this last point has a special interpretation when it comes to *ABM* economies. For many empirical quantities in economic data are distributed with heavy-tails. Example of this include Pareto distributed wealth, firm sizes, and city sizes, and so on. So skew are these data that for small numbers of agents it is hard to assess the character of the distributions that arise. With firm sizes there is a big difference in going from 10,000 agents, in which the biggest firm might be size 100 (1% of the population in the largest firm), up to 1,000,000 agents and a size 10,000 firm. Clearly an economy with a biggest firm of size 100 is quite different from one in which it is 100x larger.

C Challenge: Software Tools that are Easier to Use

Existing platforms and frameworks for building *ABMs* have been summarized above. But are these really the right tools for the economics profession today? That most of the professoriate has spent more time studying real analysis than computer programming suggests that they are not. Perhaps there is a good opportunity to create economics-specific *ABM* tools and technologies, to facilitate the creation of the kinds of models that economists care about, involving purposive agents, profit-seeking (*CES* or Dixit-Stiglitz) firms, central banks that have the Taylor rule in their behavioral repertoire, and commercial banks that lend only to businesses. In the same way that specific statistical software packages grew up to service the needs of econometricians, will new *ABM* packages appear geared toward economists who are more interested in getting a model up and running than in debugging it at code level, optimizing it at the assembly level, and so on. Almost surely such software tools are on the horizon. It us now up to us, the user community, to shape those in ways suitable for both research and teaching. Specifically, would there be significant demand for modeling tools that completely obviated the need for programming, which had an economics-specific drag-and-drop interface? It would be a challenge to create such tools, but probably worth a try!

D Challenge: Parallel Execution

One of the 'dirty little secrets' of agent computing, alluded to briefly above, is that while the speed of modern computing hardware gives users the impression that *ABMs* resemble the real-world in terms of agents engaged in joint exploitation of their local economies, there is a sense is which this is very far from the truth. For in reality it is probably the case that 99% of *ABMs* are single-threaded, meaning that there is usually only one agent active at a time, possibly interacting with another agent, but no true parallel execution. It is as if the entire world is frozen when each agent acts, as if each agent takes its environment as fixed and unchanging. Clearly this is very far from how real human social processes work. Is this a problem?

It would be nice if it were *not* a problem because to enter into the vagaries of parallel computing is not something to be undertaken lightly, and economists have no comparative advantage in writing parallel code. The good news is that in a wide variety of circumstances there is almost certainly no problem with single-threaded code, as when artifacts of execution order can be suppressed by sufficient randomization and so on, as discussed in section III.C.2 above.

But the real social and economic worlds *are* parallel and asynchronous and it is an open question as to whether there are things that can happen in such worlds that are either difficult or impossible to faithfully represent using the kinds of parallel computing tools we have available to us today. Specifically, are there states of parallel, asynchronous *ABMs* that can only be reproduced in single-threaded code by writing behavioral rules that look utterly unlike what people do in the real-world? If so, this would be a strong argument *against* single-threading and in favor of parallel execution models. Today we do not understand much about this, but it is clearly something we should know more about. The main motivation for parallel computing today is instrumental: on a machine with N cores can an *ABM* be sped up by anything like a factor of N simply by executing the model on some number of threads, $M \geq N$? Systematic exploration of such topics has begun within the *ABM* community, e.g., for the *ZI* trader model described in section II above (McCabe et al., forthcoming), but much work remains to be done.

E Opportunity and Challenge: Is *ABM* a Kind of Nanoeconomics?

More than 30 years ago the late Kenneth Arrow, in commenting on remarks of certain economic historians that seemed to be at a level of analysis 'lower' than conventional microeconomics, wondered if a new kind of economics was needed to properly address those concerns, what he termed 'nanoeconomics' (Arrow, 1987: 734). We have seen above that there may be certain aspects of *ABM* that go 'below' standard microeconomics, such as the order of execution in single-threaded models, parallel agent execution, and related issues having to do with how and when agent states are updated. Such considerations manifest themselves to some extent in the theory of oligopoly, with 'first-mover advantage' and so on, and to some extent in certain kinds of games, but are not present in other branches of micro. Largely this is because microeconomic models are solved for equilibria, with the paths along which the agents might move toward such configurations left not theorized.

This leads one to wonder if one upshot of *ABM*, in which the details of agent-agent interaction matter, may require moving to a more fine-grained nanoeconomic level. The usual micro-macro distinction in economics refers to the level of abstraction in terms of agents—traditionally macro abstracted from individuals. But with micro-foundations of macro now the norm, this distinction has been muddied. Macro today is primarily concerned with aggregate economic variables, produced by individuals. So perhaps *nanoeconomics* contrasts to microeconomics in similar fashion. We seek nanoeconomic foundations for micro in the sense that the detailed interaction histories—the nano-level—determine the micro-level. In the end, for nanoeconomics to become a field it would be necessary for it to find a home in other branches of economics. How might 'new new trade models' unfold when one exporter moves first? How can a developing country 'catch-up' to a neighbor when the neighbor has modernized first. These are the kinds of questions economists working in trade and development might care about and today there is no very satisfactory way to pose them because microeconomics does not operate at levels that would seem to be needed to resolve them.

F Opportunity: New Kinds of *ABMs* for Economics and Finance

Some 25 years ago the science journalist Waldrop thought researchers at the Santa Fe Institute were on the verge of producing 'economies under glass' (Waldrop, 1992), in software. This vision has not really been realized. To do so would require new kinds of models. For instance, imagine a model of a developing country in which all citizens and firms are represented, and with which alternative government policies can be experimented with, digitally. Or take a domain of environmental economics, say forests, and create a population of agents exploiting it. With rich models of their incentives it would be useful from a policy perspective to be able to realistically vary the rules and regulations for exploitation and see how the agents respond.

Or take the new new trade theory (Helpman, Melitz and Yeaple, 2004), written as it is at the level of heterogeneous firms. Imagine an *ABM* in which there are two countries engaged in trade with one another by virtue of the individual firms importing and exporting. We suggest that a working model in which realistic levels of trade emerge, along with credible fluctuations, would be very useful in understanding the key aspects of international good flows, and could serve as a pedagogical device.

One can imagine many more *ABMs* of that could be built but have not been. Whether to illustrate neoclassical results or to push the limits of the research frontier, such *ABMs* are possible today by the right team with sufficient resources and imaginations.

G Challenge: The Curse of Dimensionality

Models in economic theory tend to be quite minimal, of necessity if they are to be analytically tractable. But we have seen that such tractability is not a concern with *ABM*. Rather, tractability is a rationale for moving from mathematical models to *ABMs*. Thus, while a microeconomic model on some topic may have just a few parameters, many of which can be explored systematically, it is often the case that a corresponding *ABM* has more parameters. Many of the software platforms and frameworks have special tools for studying the effect of parameters on model output, such as *NetLogo's* BehaviorSpace facility, so simply having more parameters is not automatically a problem.

However, as the size of the parameter space grows the cost of exploring it rises exponentially. This is the so-called 'curse of dimensionality' first adumbrated by Bellman (1957) in the context of dynamic programming (*DP*). This problem is by no means unique to *ABMs* and whether or not it is a problem has probably as much to do with the model builder and her/his appetite for adding new parameters as it does with the domain under study. Certainly there are examples of very minimal *ABMs* having only a few parameters, exhaustively studied, e.g., segregation (Schelling, 1971a), *Sugarscape* (Epstein and Axtell, 1996), Axelrod's (1997b) culture model. There are other examples of *ABMs* having a great many parameters, many of which can be tied down with data, including power market models (Nicolaisen, Petrov and Tesfatsion, 2000).

Some critics of *ABMs* seem to believe that high-dimensional parameter spaces are characteristic of the approach, e.g., Blume (2015), but surely this is not the case. For *ABMs* are often built of completely conventional, models (Axtell, 2000), meaning that they have no more parameters than the corresponding mathematical ones.

H Challenge: The Forecasting Problem for *ABMs*

In conventional macroeconomic models the ability to create forecasts is an essential feature that makes them useful to policy-makers. Such models are estimated with current data and then alternative policies are studied, usually computationally. In considering how an *ABM* macro model might be exercised in a comparable way, an important question arises that has no analog in normal macroeconomics. For conventional macro-models feature aggregate *variables* that, at least notionally, coincide with the acquired data about the economy, so those data can be directly used in the models. But with agents, if one has only aggregate data to use it is not clear how to instantiate an *ABM* in such a way that, when it is fired up, it produces meaningful results right away. That is, *ABMs* and other simulation models often experience long 'burn-in' periods as the behaviors in the model modify the initial conditions to produce something closer to states that are consistent with the behavioral specifications. Only after a quasi-consistent steady-state is reached is the output from such models typically treated as relevant to the questions being asked—the initial transient is discarded as simply an expression of our ignorance about the true initial condition. For example, consider the Baltimore-Washington housing model described above, where households were matched to homes and mortgages. Because we did *not* have matched data at the individual level we could not start the model off in a completely consistent fashion and so typically let the model run for 50-100 years (!) to produce a state that was more or less commensurate with the agent behavioral rules. Only after this 'burn-in' period passed did we start to 'drive' the model with the real interest rate and mortgage 'innovations' that characterized the 2000-2009 price run-up and bubble burst.

In order to shrink this 'burn in' period it would be useful to know how to instantiate an *ABM* economy to minimize inconsistencies. This is the 'forecasting problem for *ABMs*'. Notice it is not solved by micro-data. That is, even if almost everything is known about the people we are modeling, it is never possible to know everything, so there will always be some inconsistency. When the micro-data is rather sparse perhaps maximum entropy or other techniques that can be used.[35] This is an open research question.

[35] Other aspects of forecasting social systems have been addressed by Hofman, Sharma and Watts (2017).

I Challenge and Opportunity: How to 'Grow' *ABM* Community Models?

With the rise of the digital computer over the past 70+ years, scientific disciplines have institutionalized its use in different ways. While computing resources were initially operated in centralized fashion within most colleges, universities, think tanks, government laboratories, and other research institutions, the personal computer revolution of the 1980s largely decentralized computing, putting it close to the ultimate users. However, in a few fields things evolved somewhat differently.

Weather modeling is one of the important legacies of John von Neumann's early efforts with digital computing (Edwards, 2010). In the wake of progress, work on weather at Princeton became institutionalized at the Geophysical Fluid Dynamics Lab (*GFDL*), supported by strong Federal government funding to keep the nascent numerical weather modeling enterprise alive. One rationale for this funding was that accurate weather forecasts were viewed as a military asset. Over decades there grew up at Princeton, and later at the National Center for Atmospheric Research (*NCAR*) in Boulder, Colorado, a suite of so-called *community models* relevant to weather, climate, and other atmospheric and oceanic projects. These models today continue to be cared for and fed with Federal research funding—the National Oceanic and Atmospheric Administration (*NOAA*) has a line in its budget for *GFDL* and *NCAR* is a Federally-funded research and development center (*FFRDC*) operated by *NSF*—to the tune of some $40M and $200M per year, respectively. Much of this money is spent on high-performance computing (*HPC*). Combining these numbers with the $1.1B spent by the National Weather Service (*NWS*), also under *NOAA*, gives some sense of the kind of support it takes to mount a scientific enterprise at continental scale. By comparison, the *entire* budget of *NSF*'s Social and Economic Sciences (*SES*) Division, within the larger Social, Behavioral & Economic Sciences (*SBE*) Directorate, is some $100M annually, Economics receiving about $30M.

Turning economic modeling into more of a community activity will perhaps take a generation, as it did with numerical weather and climate models. Figure 6 portrays the evolution.

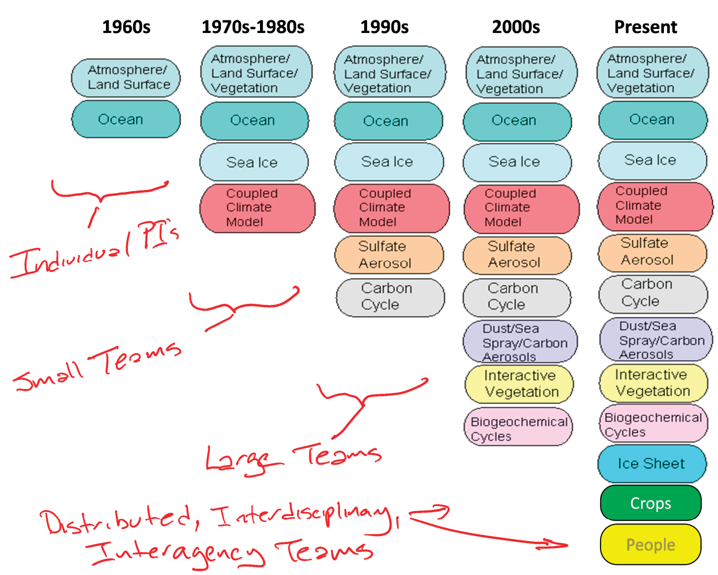

***Figure 6**: Growth of community models in weather and climate research; source: Higdon et al. (2016)*

If *HPC* technologies are going to come to the economics profession, whether in the guise of large-scale (e.g., full-scale) agent computing or more conventional numerical economics, we suggest that the model for husbanding compute resources and building scientific constituencies that atmospheric and oceanic scientists have evolved may work well for economics and finance.

V Conclusion: *ABMs* as an Emerging Methodology for Economists

Computational economies composed of software agents represent a new paradigm for economic research. Their role in substantive model building is yet nascent with some clear successes, as in finance. Many areas are relatively untouched by this new technique.

A *ABM* as a Modern Computational Methodology for Economics and Finance

By now it is hopefully clear that the strengths of the multi-agent systems methodology for economic model building involve its expressiveness (through behavioral rules not rationality assertions), its ease of implementation technologically (via software objects), its natural visual prowess (especially for communicating results), its extensibility (as the user community learns how to share and extend models), its ability to let macroscopic structures emerge (no need for pre-specification of what will happen), and its agnosticism toward agent-level equilibrium. These many features of the methodology conspire to produce great potential for relaxing one or more of the stringent mathematical specifications that plague the formal social sciences today—agent homogeneity/representative agents, rationality, indirect interactions, equilibrium.

Perhaps these many features of *ABM* methodology can serve all branches of the economic profession, from the behavioral economist's desire to better represent human behavior, to the applied economist's work to more realistically specify models and test them with data, to the theorist's focus on basic insight from abstract, stylized models.

B *ABM* as Analogous to Earlier Methodological Evolutions in Economics

The recent development of *ABM* and its application to problems in economics and finance is broadly comparable to developments in other areas of economics, some closely related. We discuss these presently.

The appearance of von Neumann's and Morgenstern's monograph (1944) opened up a whole new field in economics for immediate exploitation in the post-WWII era.[36] A bevy of important game theorists came through the Princeton Mathematics Department at this time, including John Nash, Lloyd Shapley, Martin Shubik and later Robert Aumann and Alan Kirman. Initially, none of these people got jobs in economics departments, so far afield did mathematical game theory appear to be from economics proper. It was not until the 1980s that game theory really took off within economics, primarily because models of antitrust behavior led to its widespread application by the Department of Justice. By the 1990s game theorists were actively recruited by economics departments, representing more that a 40 year lag from the birth of the discipline to its widespread adoption, and this for a methodology that was, in the main, mathematical, like the field it was trying to penetrate, although it is certainly true that there are significant differences

[36] Mirowski (2001) recounts how early mathematical economists were nonplussed by lectures given by von Neuman at Chicago in May of 1945.

from the smooth, real analysis useful for general equilibrium analysis and the more discrete, combinatorial structures one typically encounters in game theory.[37]

The situation in behavioral and experimental economics is not unlike that in game theory, with something like a 15-20 year lag. The initial results in these fields began appearing in the late 1950s and it was not until the 1990s that behavioral economics began to be accepted in mainstream economics departments, with hiring really taking off only in the 2000s, again some 40+ years after inception.

Figure 7 is a plot of the number of published papers over time that make overt use game theory as a main methodology (green), experimental techniques (magenta) and *ABM* (blue). The figure on the left is in linear coordinates and an exponential takeoff for each is apparent. The figure on the right is in *log* coordinates on the vertical axis and the roughly straight lines indicate exponential growth. Note that the takeoff phase for *ABM* has occurred with smaller lag from inception than in either of these other areas.

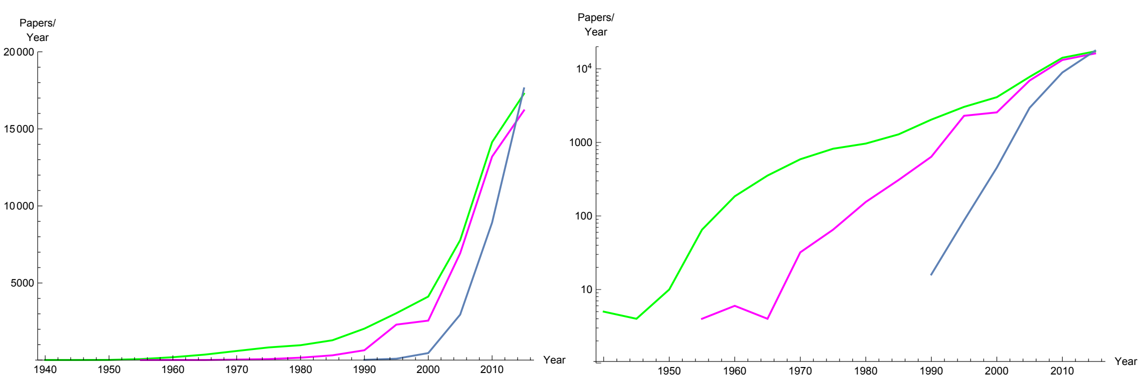

Figure 7*: Number of papers appearing each year that explicitly identify as using game theory (green), experimental economics (magenta), and ABM (blue) methods, (a) linear and (b) log coordinates*

Clearly *ABMs* resemble these other areas with a lag. Given these patterns it would seem reasonable to expect continued penetration of agent techniques into economics.

C Computational Progress in Other Areas of Science

We are living through a computational revolution. We have already compared advances in computational economics to those that are occurring in nearly every other branch of science. Earlier in this section we made explicit comparisons to numerical weather and climate computing. Among the most ambitious computational models to date are so-called 'general circulation models' used by climate scientists to forecast the likely effects of Earth's climate of human-produced emissions of (fossil) carbon, primarily from combustion. These models are written at Earth-scale and include atmospheric, oceanic, and terrestrial zone, each disaggregated into millions of discrete bins. Figure 8 gives results from the Coupled Model Intercomparision Project (*CMIP*). Each colored circle is a model and lower values of the I^2 metric are better. *CMIP*-1 goes back more than 20 years, *CMIP*-2 about 15 years, and *CMIP*-3 about 10 years. It is clear that progress in climate modeling has occurred over time. Better hardware and deeper understanding of the science involved drives this progress, while the community structure of the models facilitates it.

[37] Mirowski (2001) reports that in von Neumann's diary there is an entry that suggests Samuelson was hostile to game theory and Neumann did not expect Samuelson to ever come around.

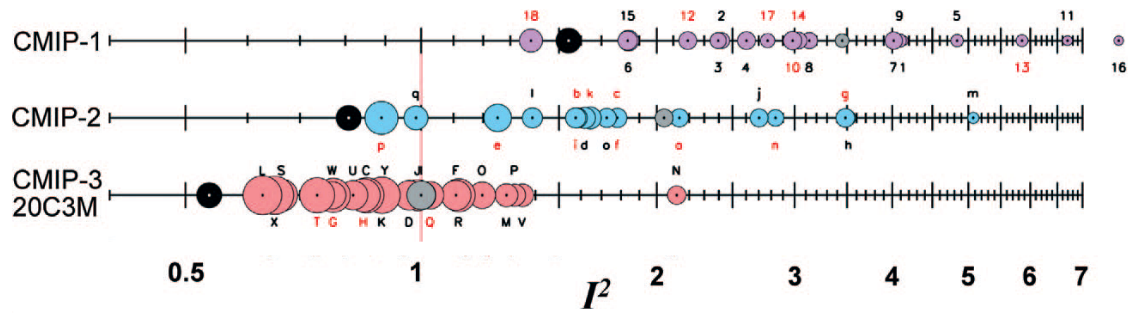

Figure 8*: Increasing accuracy in climate modeling; CMIP-1 is from the 1990s, CMIP-2 dates from the early 2000s, while CMIP-3 c. 2007; each colored circle is one model; importantly, the multi-model mean outperforms any single model; source: Higdon, Axtell, Balaji, Buja, Calvin, Carley, Castano, Coifman, Ghattas, Hansen, Michalak, Shekhar and Wang (2016)*

D Computational Economics and the Economics of Computation

Moore's law amounts to the statement that microprocessor hardware capabilities grow exponentially with time. This is largely independent of how performance is measured, whether in CPU frequency, floating point operations/second (FLOPS), transistor or memory density, hard disk capacity, or external communication rates. In most of these dimensions, performance has doubled each 18-24 months over the past two-three decades, although along some dimensions progress has stalled, leading to multi-core *CPUs*. These developments are depicted in figure 8.

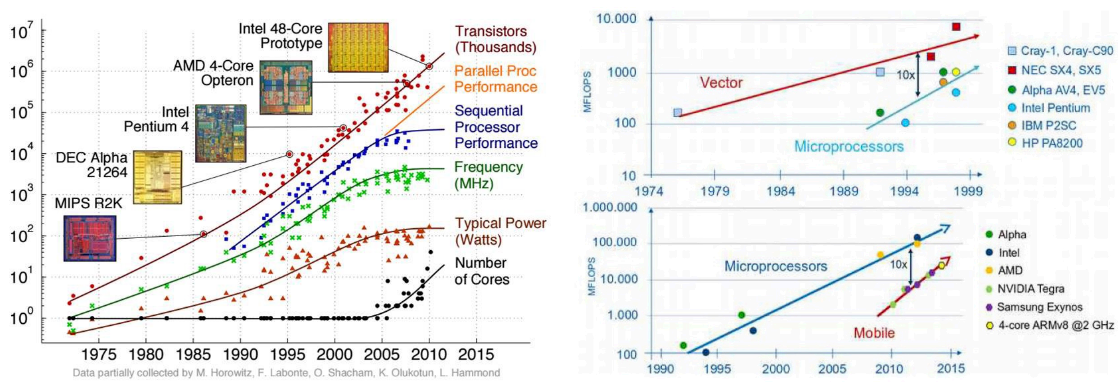

Figure 9*: Evolution of computer hardware with total transistors following Moore's law, with the performance of individual processors plateauing and greater overall performance achieved with more cores (right); convergence of commodity computing to high-performance computing over time*

A microeconomic way of describing these developments is that the cost of a unit of computation has fallen exponentially fast for a generation. As the price of computing falls economists will use more of it, unless it is an inferior good!

We have no doubt that as computing technology continues to rain down like manna from heaven, researchers in economics and finance will make progressively more and more use of it, for numerical economics, *ABM*, *SNA*, machine learning, high-frequency trading, and so on. Indeed, how long until we are managing our computational models form our mobile phones! Economists whose only substantive use of their laptop is for editing *LaTeX* documents will become a thing of the past. Papers in *Econometrica* that advertise their computing prowess in terms of *MatLab* and *Macintosh* laptops (e.g., Robin, 2011) will cease to exist. We are well on our way toward what we might reasonably call 'computationally-enabled economics' (Axtell, 2008). Indeed, the biggest challenge that researchers would seem to face today may be, 'What can we possibly achieve, on the modest hardware we have at our disposal, that will be of interest to future generations of scholars who have 10x or 100x or 1000x more?' The risk of computational economics today is not that it is being adopted too quickly, potentially over-running conventional, analytical approaches, but rather that it is being adopted so slowly that as the price continues to fall we will be deluged with the stuff without knowing fully how to use it. To a first approximation the cost of updating a single agent in an *ABM*—the marginal cost of agent computing—is falling toward zero. Will this lead to changes in the economics profession the way the Internet, with its near zero marginal cost of distributing email, news, magazines, books, images, money, control, security, surgery, even life-and-death communications, has changed all of our lives in the past two decades, for both better and worse? Time will tell.

Among the many 'flavors' of computational economics, *ABM* is poised to utilize the manna from heaven. *Numerical methods in economics* do not really take full advantage of the exponential growth in computing hardware. Greater compute power means that natural scientists can model larger systems or build models of specific phenomena at higher spatial and/or temporal resolutions. Numerical economics, along with its close cousins computational econometrics, computable general equilibrium modeling, and even microsimulation, typically do *not* fully utilize all the parts of the machine, i.e., they may use the greater processing power of *CPUs*, even *GPUs*, but typically do not utilize that enormous amounts of *RAM* that can be addressed today, or they use large-scale storage but make no use of the tremendous visualization capabilities that modern workstations often possess. Or perhaps they fail to utilize novel high-performance computing architectures (e.g., parallel execution or clouds). Agent computing techniques, on the other hand, permit the complete utilization of all extant hardware—*CPUs*, *GPUs*, *RAM*, disks, displays with millions of color pixels and palettes, fast networks, all cores, etc.[38]

One of the main results of financial economics concerns portfolio diversification. Under standard Markowitz portfolio criteria, assets with low correlation to the rest of the portfolio are assigned high weights as long as their mean expected return is positive. Similar reasoning should apply to the research portfolio of economics. *ABM* certainly satisfies the criterion of low correlation to standard methods, and (perhaps more controversially) has a positive expected mean return, and should thus play a larger role in economic research in the future.[39]

Between ever-increasing computer power to execute agent-based models, the new availability of micro-data to parameterize such models, and ongoing advances in behavioral and experimental economics to provide rules of behavior for the agents in *ABM*, the time is ripe for the field of economics to embrace agent computing as another tool in its quiver as it tries to solve hard problems associated with complex economies.

[38] This point has been made at length elsewhere (Axtell, 2008).

[39] This point is reinforced by sociological studies that show that a diversity of viewpoints results in better solutions (Page, 2007).

Appendix 1: List of Acronyms Used

ABE: agent-based economics
ABM: agent-based modeling or agent-based model
ACE: agent-based computational economics
AI: artificial intelligence
ALife: artificial life
BR: bounded rationality
CA: cellular automaton or automata
CAS: complex adaptive system
CES: constant elasticity of substitution
CGE: computable general equilibrium model
CRISIS: European Union funded project to model the Financial Crisis with *ABM*
DAI: distributed artificial intelligence
DSGE: dynamic stochastic general equilibrium, the conventional approach macro models
EU: European Union
EWA: experience-weighted attraction, an empirically-grounded learning algorithm
GFDL: Geophysical Fluid Dynamics Laboratory at Princeton University
GIS: geographic information systems
GSIA: Graduate School of Industrial Administration, first at Carnegie Institute of Technology and later Carnegie-Mellon University; today: Tepper School of Business
IBM: individual-based model
LANL: Los Alamos National Laboratory
LFN: labor flow network
MAS: multi-agent systems
MERS: Middle East Respiratory Sickness
MIDAS: Models of Infectious Disease Agent Study at *NIH*
MLS: Multiple listing service, a real estate firm and data aggregator
MRS: marginal rate of substitution of one good for another
NASDAQ: National Association of Securities Dealers Automated Quotations
NCAR: National Center for Atmospheric Research
NIH: National Institutes of Health
NOAA: National Oceanic and Atmospheric Administration
NSF: National Science Foundation
NWS: National Weather Service
OFR: Office of Financial Research within the Department of Treasury
OR: operations research
SARS: severe acute respiratory syndrome
SBE: Social, Behavioral & Economic Sciences Directorate at *NSF*
SD: system dynamics
SES: Social and Economic Sciences Division at *NSF*
SNA: social network analysis
SOES: Small Order Execution System on the *NASDAQ*
UCAR: University Consortium for Atmospheric Research
WMAD: Walras-McKenzie-Arrow-Debreu model of general equilibrium
ZI: zero-intelligence, trading agents who act purposively but without an internal model
ZIP: zero-intelligence plus trading agents

Appendix 2: List of Computer Terms, Languages, and Systems Mentioned

ACT-R: cognitive architecture used in some agent systems
ASCII: American Standard Code for Information Interchange, for character encoding
BASIC: early programming language, little used today
BDI: belief-desires-intentions representation of agent behavior, popular in *MAS*
C: early low-level programming language, still in wide use today
C++: object-oriented version of *C*
C#: object-oriented programming language from Microsoft
CMIP: Coupled Model Intercomparison Project
CPU: central processing unit
DP: dynamic programming, pioneered by Richard Bellman in the 1950scu
DSGE: dynamic stochastic general equilibrium model of macroeconomics
EINSTEIN: combat modeling toolkit
EPISIMS: epidemic simulation code derived from TRANSIMS at Los Alamos
EurACE: agent-based macroeconomic model in use in Europe for research and policy
FORTRAN: early programming language, still in use today for scientific computing
GAMS: General Algebraic Modeling Systems
GEMS: General Electric modeling and simulation language
GPSS: general purpose simulation system
GPU: graphics processing unit
HPC: high-performance computing
ISAAC: Irreducible, Semi-Autonomous…
JABOWA: early forest simulation system in *IBM* ecology
Java: object-oriented programming language originally created by Sun Microsystems, currently owned and operated by Oracle
MASON: agent-modeling software framework in Java from George Mason University
NetLogo: popular *ABM* environment requiring modest programming background
NP: complexity class of problems solvable nondeterministically in polynomial time
Objective-C: early object-oriented programming language, still in use at Apple
ODD: protocol for reporting *ABM*s
OOP: object-oriented programming
P: complexity class of problems solvable in polynomial time
PAC: probably approximately correct learning, a learning algorithm
Pascal: programming language created at ETH Zurich in the 1970s, litte used today
PPA: complexity class between *P* and *NP*; stands for ‘polynomial parity argument’
RAM: random access memory
RePast: agent-modeling software framework in Java and C# from Argonne
RNG: random number generator
SimScript: early simulation language, still in use today
SIMULA: the first OOP language and a family of simulation languages
SmallTalk: early object-oriented programming language, in little use today
SOAR: early cognitive architecture
StarLogo: early programming language for beginners from MIT
Sugarscape: early *ABM* in which agents forage for resources and engage in exchange
SWARM: early agent-based modeling language
TRANSIMS: transportation simulation code created at Los Alamos

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